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Front and back covers: Photomontages by Harry Sowden of Centre Beaubourg and Sydney Opera House newscuttings

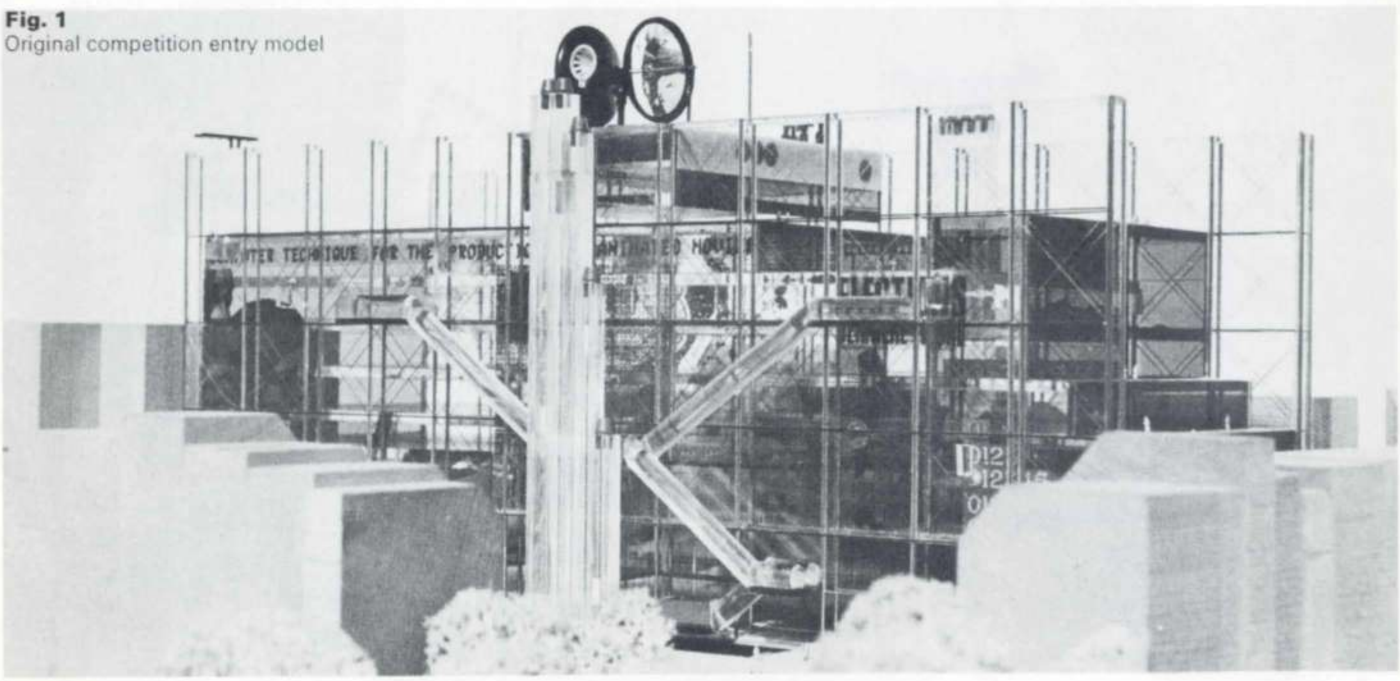
Centre Beaubourg: Introduction

Ted Happold
Peter Rice

The competition was announced early in 1971 in the *RIBA Architectural Competitions Newsletter*. In 1970 Richard and Su Rogers had contacted Frei Otto for a project in Chelsea and he suggested that they engage Ove Arup & Partners Structures 3. The competition offered an ideal opportunity to extend the relationship and we asked them if they would be interested in entering with us.

The Rogers brought along Renzo Piano, an Italian architect, with whom they were merging their practice. A series of meetings was held to decide whether to enter, which were always spent defining the answers we wanted. The competition brief was good and the French love of originality, together with the list of names of the judges which included Philip Johnson, Jørn Utzon, Oscar Niemeyer and, as chairman, Jean Prouve, ensured integrity and recognition of the correct solution. It was also a good time to put in a British entry as both the Burrell Collection Competition and the Houses of Parliament Competition were in progress. It was felt that all good designs have a very strong pedestrian movement plan. The answer was an information centre which was alive for people rather than a spectacular monument. Adaptability was seen as essential and servicing zones provided for this. The structure was seen in the tradition of Eiffel – the modern equivalent being the main space frame roof for Osaka with its cast steel nodes being machined for connection. The size of the building was determined by the very high plot ratio.

Fig. 1
Original competition entry model



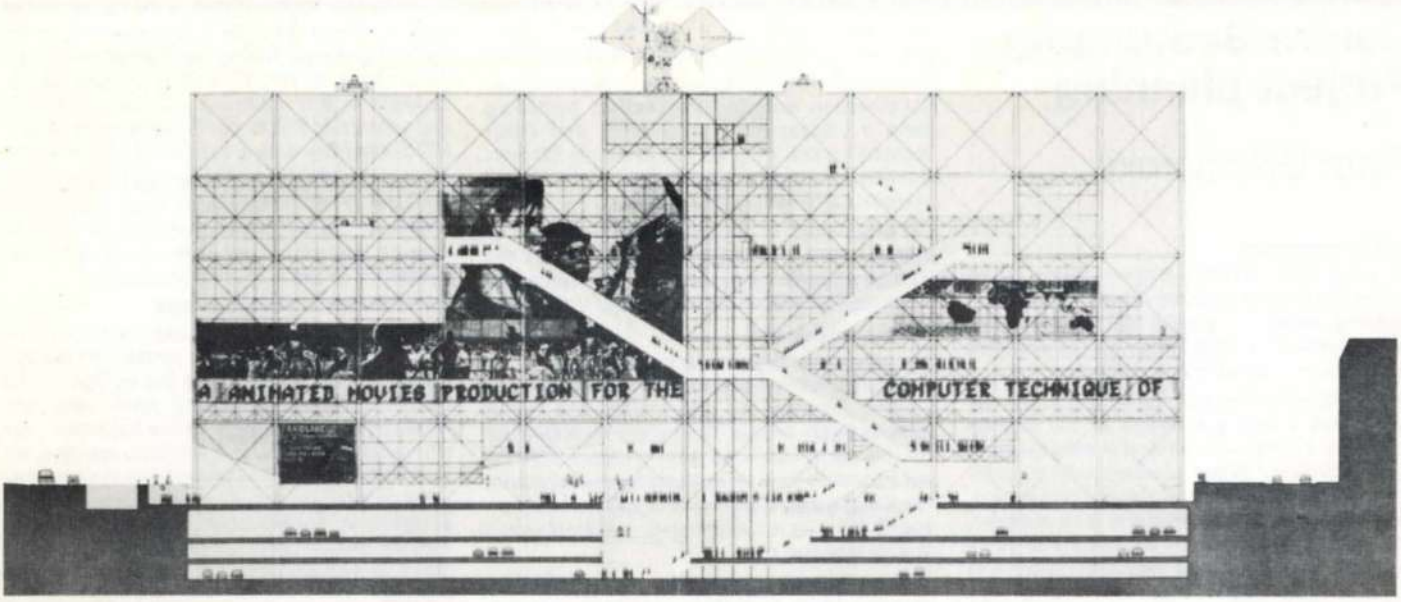


Fig. 2
Original competition entry elevation

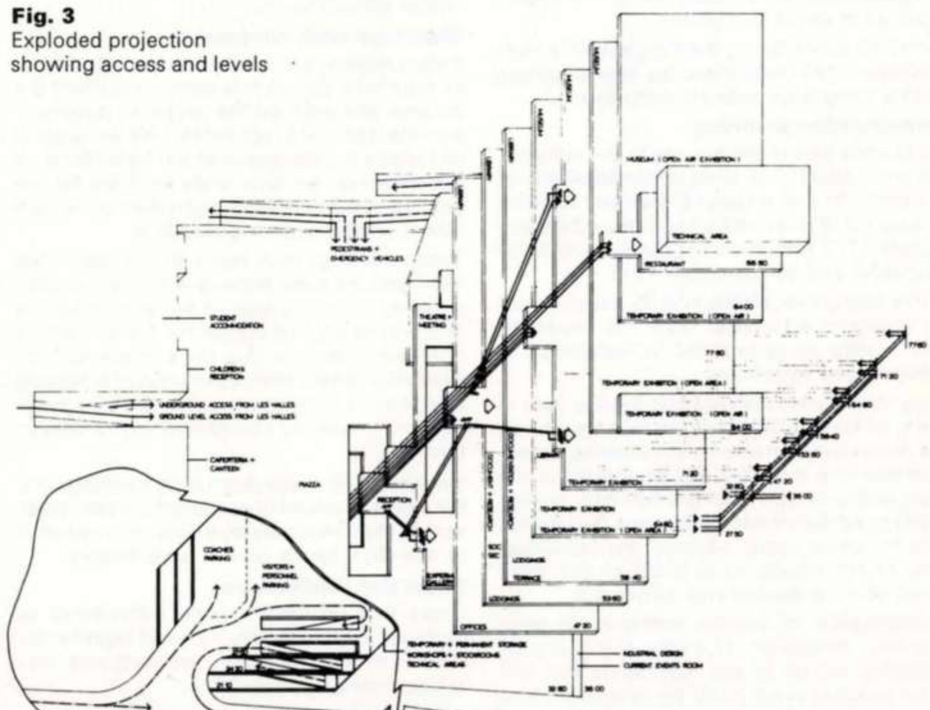


Fig. 3
Exploded projection showing access and levels

The entry defined the centre as a cross between an information-orientated, computerized Times Square and the British Museum, with the stress on two-way participation between people and activities/exhibitions.

Information was planned to be shown in three areas:

(i) Within the building, which offered a number of large, adaptable, uninterrupted floor areas for the library, cinema and exhibition of fine arts, architecture, etc.

(ii) On two façades, one facing west across a sunken square and the other facing a high-speed street at the rear. Three-dimensional load-bearing walls carried constantly changing information. The façade facing the rear was planned to have visual displays related to moving traffic, whilst the façade facing the square related to pedestrians.

(iii) The large sunken square and the area around the edge of the square was to provide horizontal continuity to the façade to include exhibitions, parades, etc.

In order to segregate pedestrians and vehicles, a pedestrian sunken square was formed 3.2 m below ground level, linking through at low level to the surrounding pedestrian routes. The car park for visitors and personnel was on three floors beneath the square. For easy orientation and flexibility the main vertical movement in the building was planned to take place on the face of the building, so that it could be clearly seen by anyone viewing the building from the square in front. The only built form which interrupted the open square was the glazed reception area under the free-standing building. The reception was vertically linked by escalators and lifts, down through a wide opening in the square to the parking below and up the face of the building to the different departments.

On 18 July 1971, the competition was announced and an exhibition opened at the Grand Palais. The total number of entries was 681 and while no second prize was given, 30 recommended prizes were awarded, one with a distinction. The design with a distinction was an even more negative architectural building than the winner – a several-storeyed concrete flat slab building covering the entire site with largely undefined areas ('museum corner', etc.) and a similar information wall at one corner. Our group was influenced by Venturi; the second group were students of his. Many entries were incredible – monumental buildings based on such simple everyday forms as coiled springs, etc.

There was immediate pressure to continue the design, as the Government wishes the building to be completed by November 1975. The group set up to run the competition continued as the client body and Piano + Rogers and Ove Arup & Partners were appointed on a three-month study contract to amend the design, taking into account developments subsequent to the competition brief. For the first month a design office was set up in the Grand Palais and then moved to a building by Place St Victoire. The revised design was submitted in October. The changes were quite great and for the first time the design team was told a cost objective. A fairly radical redesign took place, probably going more back to the spirit of the original competition entry. Meanwhile the lease on the Place St Victoire ran out and the team moved to a 'gonflab' on the Quai by the Seine opposite Ile St Louis. Whilst beautifully situated, the pneumatic was too noisy and too small and in the summer a longer lease was taken on the present offices in the Rue Reaumur.

The development of the design since the competition has gone through many phases and, as with the seven ages of man, it has grown and become more explicit with time. The search was for a design which did not dominate the activities and, given the scale of the building, this was not entirely easy. Flexibility as a concept was replaced by adaptability. The movable floors and the mechanical connections which they generated were replaced by the concept of a single large space. This space can be adapted and changed with use. An early solution to this problem of large open spaces was to make a vierendeel structure with alternative structural and non-structural floors. This gave a layered building and destroyed the concept of simple repetitive spaces. The present solution was a return to the competition entry, but the detail, achieving the same kind of complexity that the movable floor machinery would have done, went through many alternatives and it was helped enormously when the architect defined the façade and the external movement zones and the carrying on of a prejudice that cast steel was the correct material to use, stemming from a relationship with Koji Kamiya, the architect for the Osaka roof, and confirmed by a trip to Japan.

Centre Beaubourg: Project planning

Peter Bolingbroke

Initial appraisal

In June 1971, Piano+Rogers asked us to assess the likely construction time for the main building, based on their outline design proposals, since the competition rules required this information to be submitted with each entry.

Our appraisal of the scheme at that time indicated that a contract period of 52 months should be allowed for carrying out the building works involved. When coupled with an estimate of the time required to develop the design, obtain competitive tenders and let a contract, an overall project time of six and a half years resulted. This information was duly included in the architects' report when submitting their entry.

Shortly after the result of the competition had been announced, it became apparent that the organizers required the Centre to be opened to the public before December 1975 – i.e. within a period of about four years!

A major problem facing the design team in mid-September 1971 was, therefore, how to achieve such a completion date for the project.

Construction planning

To provide part of the answer to the problem, we were asked to investigate the feasibility of constructing and equipping the main building in a period of three and a half years – between 1 June 1972 (when part of the site would be available) and 30 November 1975.

Of the total time available, only 38 months could be allocated to builders' work, the remaining four months being required for installation of furniture and equipment.

Our initial studies showed that the key item in the total construction process was erection of the massive steel frame for the building superstructure. We therefore had to determine the programme for this element first and, having established the minimum time required for the superstructure work, allocate the remaining time as realistically as possible to the work items which preceded and followed it.

Consideration of various methods for steel erection, installation of floors and fixing of cladding, led us to the conclusion that the superstructure work could be carried out in a period of 14 months. On the assumption that at least a year must be allowed after completion of the superstructure for installing the M & E services and internal finishings, it was apparent that only 12 months could be allocated to the substructure works which preceded steel erection.

Having estimated the time-scales for these three major stages of construction, two programmes were prepared which were similar as regards the superstructure and finishing stages but indicated different methods of achieving the substructure stage. The first programme was based on conventional methods of carrying out the substructure works and the second illustrated a scheme whereby the ground floor slab could be cast first. We discussed these programmes with a steelwork contractor and a general contractor in France, who subsequently confirmed that the time allowed for the superstructure work was feasible but had considerable doubts about the possibility of completing the substructure, by conventional means, in 12 months.

Preliminary report

At the end of October 1971, we prepared a project planning report which included a full explanation of all construction studies carried out to date and recommended that the substructure be built by a method which gave priority to completion of the ground floor slab.

The report also drew attention to the main pre-construction activities involved in achieving such a construction programme and commented on the short period available for production of design information, ordering of materials and letting of contracts.

Consequently, it suggested that a main or management contractor be appointed at the earliest possible date to enable the design and construction processes to be fully integrated.

Main contractor

The concept of appointing a main contractor to work with the design team initially, and subsequently manage the site works, was not a familiar one in France. To explain our proposals in more detail and explore the contractual problems posed if such an appointment were made, we submitted a report on the subject and followed it up at meetings with the Delegation during November.

It was eventually agreed that a form of contract could be devised which would allow a contractor to play a management role in the execution of the site works, so a series of preliminary meetings was arranged with ten French contractors with a view to selecting a short list for further consideration.

Meetings with contractors

Before meeting the contractors we issued each of them with documents which described the scheme and outlined the proposed construction method and programme. The documents included a questionnaire which called for their comments on the time-scale available for the works and our concept of the main contractor's role in achieving such a schedule.

These meetings took place during December 1971 and we were pleased with the response obtained to our proposals for executing this difficult project. The majority of the contractors confirmed our view that the procedures outlined afforded the best opportunity of achieving design and construction within the time available and producing a building of high architectural quality.

Resulting from these preliminary meetings and the replies received to our questionnaire, it was agreed that five contractors should be included on the short list for further consideration.

Short list contractors

These five contractors were interviewed in depth by a team which included representatives from the Delegation, architect and ourselves, during January 1972.

At least one day was devoted to each contractor, to meet the personnel responsible for design work, cost control, site organization, etc. A further day was spent with each contractor visiting sites in progress and inspecting completed buildings. Arising from these investigations, we prepared a detailed analysis of the capabilities and attitudes of each contractor and submitted this to the Delegation in March 1972, together with a report on our findings.

All five contractors were invited to tender for the post of main contractor and, during the tender period, we again met each of them to answer queries arising from the information contained in the enquiry documents.

Construction programmes

Between November 1971 and the time when contractors were invited to tender, the design team had been developing the design of the building. During that period many variations were made in the design of the basement, the structural system, M & E services, etc. and, on each occasion, we had to examine the implications of these on the construction method and programme. Furthermore, we had to accommodate revised dates for possession of certain areas of the site and for completion of the project.

By the time the tenders were received from the short list of contractors, we had prepared some 15 programmes relating to the construction works.

When the successful contractor had been appointed, we joined in preliminary discussions with the personnel responsible for planning and programming the current scheme. These discussions culminated in the production of the contractor's working programme, which then formed the basis for all site work up to completion of the structural steel frame.

This programme provided for the substructure work being carried out in a conventional manner, with the lowest basement slab being cast first.

Such a basic change of plan arose from the fact that the steel-framed superstructure was finally designed to start at a level 9 m below ground level, thus reducing the amount of substructure which had to be completed prior to erection of the steelwork. The revised design also necessitated the ground floor slab being cast after erection of the steel frame and precluded the possibility of constructing it in the manner originally proposed.

Other programmes

From November 1971, we also produced a number of programmes relating to various pre-construction activities. These programmes outlined the design team's work and the letting of individual contracts for excavation, foundations, substructure and steelwork. Separate programmes covered the demolition, diversion of services and other preliminary site works which had to be done before the main construction works could commence.

Centre Beaubourg: Traffic considerations

Michael Sargent

Plateau Beaubourg lies on the right bank of the Seine and is located between the historic centre of Paris, Le Marais, and the former market area, Les Halles, which is now being redeveloped (Fig. 2). The site is bounded by Rue St Martin on the west, Rue du Renard on the east, Rue Rambuteau to the north and Rue St Merri to the south. Rue du Renard carries large traffic volumes, is one-way southbound and varies between four and five lanes in width. Rue St Martin is also one-way southbound, varies in width between one and two lanes and is considered to be an important traffic route at peak hours. Rue St Merri and Rue Rambuteau are less important traffic routes.

The Les Halles redevelopment consists of commercial and other uses served by an underground road system shown in Fig. 1. There are five entry and exit points to this underground system with full height entry points (4 m) in Rue des Halles, Rue Turbigo and Rue du Louvre. The entires from Rue Rambuteau and Rue Pont Neuf will be of limited headroom (2.6 m) for cars only.

The brief defined in the competition documents required the provision of an underground car park for 800 visitors' cars and 110 staff cars, an underground coach park for 20 coaches and facilities to unload eight lorries in an area secure from fire and possibility of theft. In addition, a design was required for the connection of the Les Halles tunnels under Rue Rambuteau and Rue St Merri with the road system on the surface.

A large number of possible schemes for access to the Centre was investigated. The final scheme was chosen after consultations with the City of Paris Planning and Engineers Departments and SEMAH (Societé Anonyme D'Économie Mixte D'Aménagement de Rénovation et de Restauration du Secteur des Halles).

The main access for cars and the sole access for commercial vehicles is from the St Merri tunnel. In the short term, before the completion of the Les Halles underground road system, entry to this tunnel will be via a temporary ramp located in Square des Innocents. A secondary entry for cars only is situated in Rue Rambuteau. Initially a ramp will be constructed leading solely to the car park. It is intended that this will form the entrance to a future west-bound tunnel to Les Halles.

The exit from the basements of the Centre will be to the continuation of the St Merri tunnel eastwards. It is proposed that this tunnel will climb from 27 m level at the exit from the basement to pass over the metro tunnel under Rue du Renard and join this road at the surface at approximately 36 m level via a curved ramp located south-east of the junction of Rue St Merri and Rue du Renard.

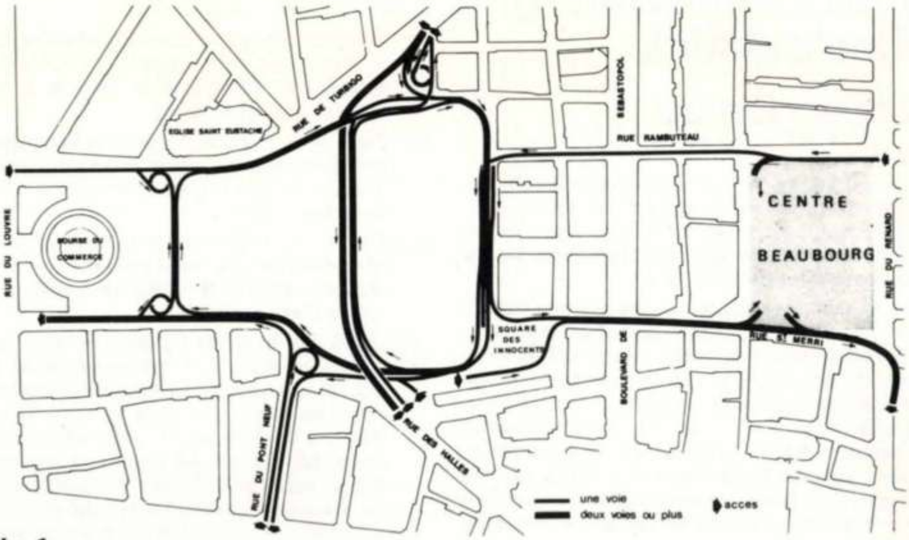


Fig. 1
Plan of underground road system

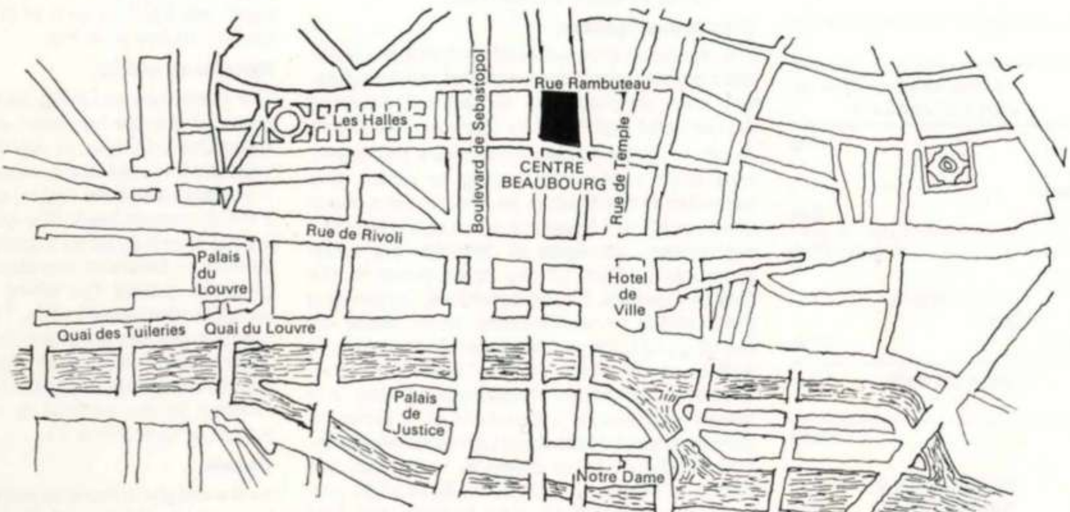


Fig. 2
Map showing the relation of the Centre Beaubourg to the historic centre of Paris.

The underground car park (Fig. 3) will be of the split level type on six half floors between levels 28.5 m and 21 m. The level of the entry and exit to the St Merri tunnel is at 27 m, while the entry from Rue Rambuteau is at 28.5 m. The split level solution was chosen because of the need to accommodate two entries at different levels and to allow for the different headrooms needed for car and coach parking while keeping the total volume of the car park to a minimum and using this volume as efficiently as possible. The car park is designed with a clear span layout and the relevant dimensions are as follows:

Bay size: 5 m × 2.4 m

Aisle width: 6.0 m

Ramp width minimum: 4.3 m between walls

Clear headroom: 2.25 m under beams.

An underground coach park is proposed at the 27 m level for approximately 18 coaches adjacent to the building. The design has been based on the manoeuvring characteristics of vehicles 12 m long.

An underground service yard is shown also at the 27 m level and north of the coach park. This yard is designed to accommodate vehicles 15 m long.

Because of the conflicting movements which will occur at the entry to the basement from the St Merri tunnel it is proposed that traffic signal controls should be installed.

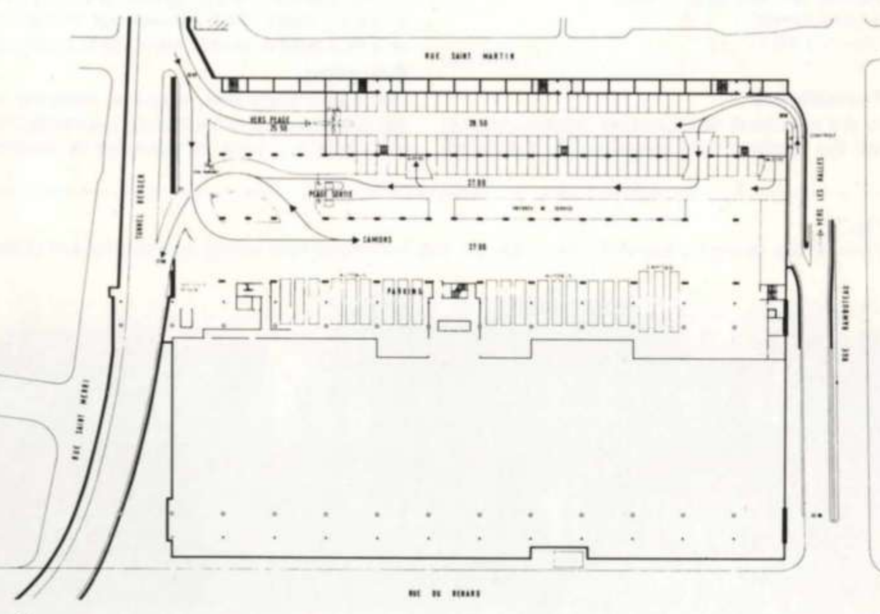


Fig. 3
The underground car park

In the scheme submitted for the competition entry it was proposed that Rue St Martin should be closed to vehicles and that the level of the carriageway should be reduced to expose the cellars of the fronting properties. This was found to be impracticable because of the sewer beneath the road. The Préfect de Police has agreed to a temporary closure of this road, after which a decision will be taken on its ultimate future.

Centre Beaubourg: Geotechnics

Martyn Stroud

Geology

The geological sequence consists of the alluvial deposits of the valley of the River Seine, overlying soils and rocks of Eocene Age. Strong correlations exist with the sequence in the Hampshire basin in England.

At Centre Beaubourg, we have been mainly concerned with the upper part of the Marnes et Caillasses, which consists of a very stiff to hard, weathered dolomitic, chalk marl from which the evaporites (such as gypsum) have been leached out. The lower part of the Marnes et Caillasses has the appearance of a shattered, open-fissured limestone, but contains near its base a hard layer of relatively intact limestone – La Rochette – which serves as an excellent founding layer.

Beneath the Marnes et Caillasses is the Calcaire Grossier, which is a moderately strong fissured limestone.

In more detail, the geological sequence at the site is:

Paris Basin	*Corresponding Beds in Hampshire Basin*	
Remblais (fill)		6 m
Alluvions modernes et anciennes (recent and ancient alluvium)		6 m
Marnes et Caillasses (dolomitic marl with calcite nodules)	Bracklesham Beds	5 m–14 m
Calcaire Grossier (massive limestone)		25 m
Sables Cuisiens (Cuisiens sand)	Bagshot Beds	12 m
Fausses Glaises ('false silt' – silty clay)		5.5 m
Sables d'Auteuil (Auteuil sand)	Woolwich and Reading Beds	3 m
Argiles Plastiques (plastic clay)		8.5 m
Marnes de Meudon (Meudon marl)	Chalk	
Craie (Chalk)		

Geotechnics

As the structural requirements became known and the available site investigation data were studied, four major geotechnical problems became evident. These were:

(a) Foundations for the main columns, unusual in that each was required to sustain a moment of 18,000 Tm as well as a net vertical load of 4,000 T.

(b) Earth retaining structures of a temporary and permanent nature required during excavation and construction of the 16 m deep basements.

(c) Heave resulting from the removal of 16 m of overburden, coupled with the swelling characteristics of the Fausses Glaises and Argile Plastique beds.

(d) Uplift due to water pressure beneath the basement slab. The basement level of +20 m was just above the normal ground water level but, in times of exceptional flood, a level of +27 m NGF was to be expected. This would result in a pressure due to water of 7.0 T/m², compared with an average structural dead load over the basement of about 2.9 T/m². In order to estimate the quantities of water that a drainage system would have to cope with, a knowledge of the permeabilities of the various beds was required.

Site investigation

While preliminary studies of each of these problems were being made, a detailed site investigation was planned and carried out between January and April 1972 by Simecsol Sondages.

In all, 17 rotary drilled holes were put down, nine with continuous coring of which two extended to the Marnes de Meudon at a depth of about 80 m. 13 borings were equipped with piezometers arranged to monitor the water pressures in each of the strata down to the Sables Cuisiens. In five boreholes, continuous permeability measurements were made by carrying out Lugeon tests between the phreatic surface and the base of the Calcaire Grossier. These tests simply involved pumping water into sealed sections of a borehole at a series of constant pressures and noting the flows for each. Two shafts of 1.5 m diameter were also put down, to depths of over 20 m. Each shaft was hand-dug and took ten days to complete. This examination of the founding materials, in situ and at depth, was invaluable to our understanding of the upper Marnes et Caillasses.

To supplement the Lugeon tests, full scale pumping tests were carried out in the shafts and in a well specially sunk for the purpose.

Barrettes

The major loads and moments from the steel structure have been founded on barrettes. These are walls 9 m long constructed in reinforced concrete in a trench which is dug under bentonite mud, in the same way as the method used for constructing diaphragm walls. The essential difference is in the scale of the concreting operation, since each barrette must be poured in one operation with a concrete volume of 200 m³, including overbreak. In general, the barrettes have been taken down to the Rochette.

The large applied moment is thus resisted in shear on the sides, ends and base of the barrette and by reaction from the lower part of the Marnes et Caillasses. An additional counterbalancing moment is applied by jacking horizontally at +27 m level.

By estimating the variation of Young's Modulus of Elasticity in the upper and lower beds of the Marnes et Caillasses and in the Calcaire Grossier, David Henkel was able to derive a number of 'spring constants' to represent the soil response around the barrette. With these values, it was possible to estimate the order of displacement and rotation of the barrette under load and to check that at no point were the allowable stresses of the soil and concrete exceeded.

As the rock strata dip gently to the north-east, the barrettes for this part of the building are taken down to a greater depth than at the south-west end. The toe level of the barrettes ranges from +10.5 m to +3 m.

Retaining walls

The temporary retaining wall around the perimeter of the site has been constructed on the Berlinoise principle, in which rolled steel sections are lowered into pre-bored holes. The toe is concreted and the upper part back-filled with a weak cement bentonite grout. The retaining wall is constructed as excavation proceeds by slotting in between the steel king piles either concrete panels (as along Rue du Renard, which is illustrated in Fig. 1) or wooden horizontal members (as along Rue St Martin). The wall is anchored by prestressed ground anchors which, along Rue du Renard, were very steeply inclined to the vertical to avoid striking the tunnel of Metro line 11.

Heave

As the weight of material excavated underneath the main building and the adjacent car park exceeds the weight of the building, long term heave will take place. In particular, the soils which are susceptible to swelling, following a reduction in effective stress, are the Fausses Glaises and the Argiles Plastiques. Calculations have shown that the final upwards movement of the basement will be acceptable and, at most, about 60 mm.

As an aid to the monitoring of heave in and around the basement during excavation and

Fig. 1
View of site showing cuvelage excavations and Berlinoise wall along Rue du Renard (Photo: Laurent Rousseau)

construction, a deep bench mark was installed at a depth of 80 m in the Rue du Renard during the site investigation. A multiple heave gauge was also installed close to the centre of the site, and this is already yielding measurements of the vertical movements of points spaced at 2 m intervals between the base of the excavation and the top of the Marnes de Meudon.

Uplift

The problem of uplift has been overcome with a system of drainage relief wells extending to a depth of 5 m below the basement.

A grout curtain which extends below the retaining wall is designed to reduce the flow of water into the basement to within controllable limits at times of exceptional flood. The curtain is formed by using two different grouts injected successively in two separate phases using the 'tube-à-manchette' method. The primary grout is a cement bentonite mixture, with the addition of a deflocculating agent. The secondary grout is a bentonite/silicate gel using a weak organic acid (boric acid) to neutralize the silicate ions to form a hard grout.

The sub-basement, or 'cuvelage', which was introduced into the building after the scheme for the temporary retaining wall had been designed and the contract had been let, extends below the existing ground water table. In order to construct this sub-basement, it has been necessary to install a system of deep filter wells to lower the ground water. The pumping from these wells has complicated the final closure of the grout curtain, since, as the uncompleted length decreases, the water flows at a faster and faster rate through the gap. At the present time, modifications in the grouting procedure are being considered so as to mitigate the effects of pumping and to ensure that an adequate grout curtain is constructed along the closure length, which will be at the western end of Rue Rambuteau.

Programme

The southern part of the site was cleared initially and, after the preliminary excavation from +36 m to +26 m level was carried out in the central part of this area, the grout curtain work began, closely followed by construction of the Berlinoise retaining wall. Excavation then proceeded to +21 m level, from which level the barrettes have been constructed. Subsequently, the filter wells were installed around the area of the sub-basement, where excavation is now in progress. The final stages of the geotechnical work follow immediately after the demolition of the buildings on the southern side of the Rue Rambuteau. These are to complete the grout curtain, as described above; to construct the Berlinoise wall; to finalize the excavation and to complete the barrettes. Concurrently with this work, the sub-structure will be brought up to a level of +27 m prior to the erection of the steelwork during the latter part of 1973.

Centre Beaubourg: The sub-structure

John Morrison

The infrastructure covers an area of 160 × 100 m and is approximately 16 m deep with a further local area for the cuvelage of 70 × 40 × 4 m which gives a total depth of 20 m.

The infrastructure itself can be divided into two main sections:

(1) Car and coach parking and general service vehicle movements zone which is situated to the west of the main building and under the piazza;

(2) The zone below the steel structure occupying roughly half the site, containing plant rooms, storage areas, theatre, cinema and art galleries, etc.

Car parking structure

The car park area consists of three bins, 16 m × 148 m. The column grid is 16 × 6.4 m, the 6.4 m being the half module of the main superstructure. The clear span of 16 m was adopted to give flexibility of car spacing; if the 16 m span had been reduced it would have led to an inefficient parking system as the 2.4 m car bay does not fit 6.4 m, the 6.4 m grid being indispensible to the architect due to the close relationship of vehicle movements and the grid of the main structure. The car park thus has main beams spanning 16 m onto columns and between these beams there is a 150 mm slab. One advantage of this system was that services could pass across the width of the three parking bins without dropping below the beam soffit.

Structures under main building

In this section a grid of 12.8 × 8 m beam and slab system was adopted, largely due to French fire regulations. It was originally intended to use a waffle slab but fire regulations recently introduced by the CSTB virtually preclude the use of waffle slab types of structure by requiring that all beam ribs have a minimum of three levels of reinforcement for a two-hour rating.

Previously French regulations have been very similar to the English, i.e. specifying minimum thickness of members and their cover.

For a two-hour rating the minimum slab thickness is 110 mm. Using this value for the flange depth and maintaining the required regulation minimum span/depth ratio of 16, the spacing of the beams was calculated as 3.2 m. This dimension also worked conveniently with various other details in this section of the work.

For the theatre, where larger clear areas are required, we have walls spaced at 25.6 m, and these have to be built after the erection of the main superstructure. For ease of construction, a Preflex beam solution has been adopted for the theatre roof.

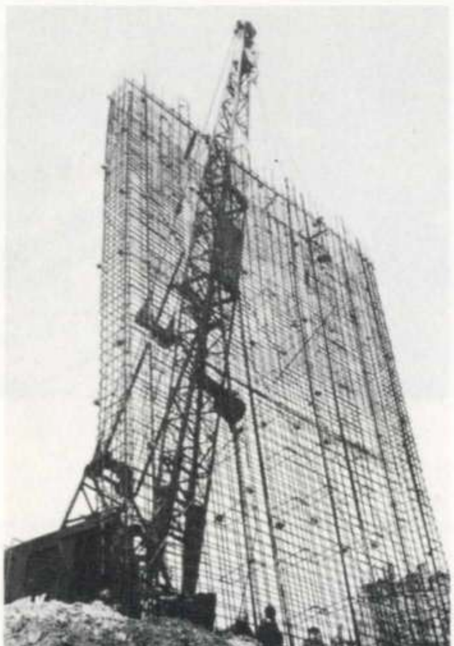


Fig. 1
14 m × 9 m × 1 m reinforcement cage being lowered into excavation to form the barrette. (Photo: Laurent Rousseau)

Contract Lot 1

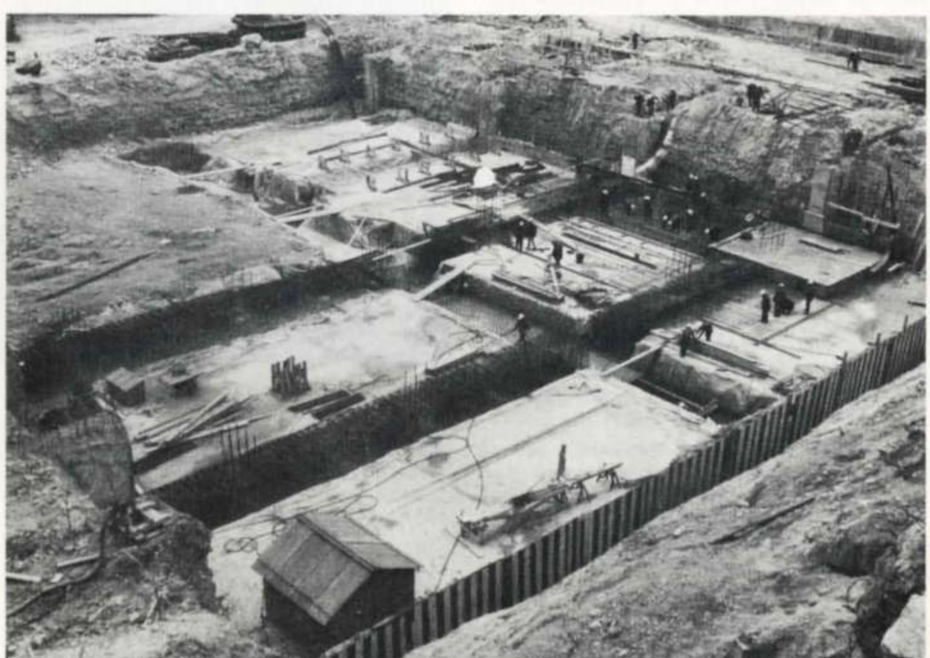
The article on project planning has explained the problems resulting from the extremely short time available for the construction of this building.

In principle it was important that a start on site should be made almost immediately if the final completion date of October 1975 was to be achieved.

By virtue of the very complexity of the building, a total contract was out of the question but various sub-contracts such as excavation could be let fairly quickly, thus gaining design time.

To ensure efficient planning and co-ordination of the sub-contractors, a management contractor had to be appointed. The direct appointment of a contractor by negotiation was impossible due to Government restrictions. It was therefore necessary for the prospective management contractors to have to tender for a small section of the work as part of the package. The removal of 6 m of ancient cellars, virtually a demolition contract, was therefore

Fig. 2
Construction of the cuvelage 20 m below street level. (Photo: Laurent Rousseau)



included. This work involved 100,000 m³ of excavation and was sent out to tender in February 1972, two months after our arrival in France.

The successful contractor was Grands Travaux de Marseille. GTM is one of the leading French contractors with associated groups throughout the world.

Lot 2 – Grout curtain

This is described in Martyn Stroud's article The contract for Lot 2 was awarded to Intrafor Cofor in March 1972.

Lot 3 – Excavation and Berlinoise wall

This contract consisted of the excavation of 160,000 m³ of gravel and soft rocks together with the provision of a temporary wall (Berlinoise) to hold back the 16 m high excavated face.

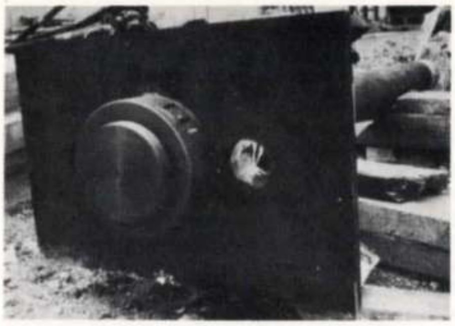


Fig. 3
Anchor plate for 1000 tonne tension in external columns of main frame work.
(Photo : Laurent Rousseau)

Lot 5 – Reinforced concrete sub-structure

This contract was sent out to tender in October 1972 so that the successful contractor could start on site in December 1972. The contract included all car parking structure, plant rooms and storage areas below the building. The target is to construct a slab at +27 m level, which will then be a suitable platform for the erection of the structural steelwork. The target date for the completion of this phase of the contract is August 1973.

After the steelwork has advanced sufficiently, the contractor will return to complete the theatre, art galleries and piazza slab, but at this stage the concrete work is no longer in the critical path. Quillery St Maur have been appointed contractors for this phase.

Fig. 4
Cuvelage and buttress for toe of Berlinoise wall
(Photo : Laurent Rousseau)

The Berlinoise wall consists of pairs of 400 mm steel sections at 3 m centres, held by grout anchors on three levels. Between these profiles is in situ concrete which was specified as part of our waterproofing system but in other situations this could have been precast concrete or wood.

This contract was also let in March 1972 to Entreprise Coutant. It was important that the installation of the profiles and the excavation followed closely behind grouting if the programme was to be maintained.

Lot 4 – Barrettes

The barrettes are the principal foundations for steel superstructure and are in fact sections of diaphragm wall, 1 m wide by 11 m long. Each barrette carries two vertical loads of 5000 tons compression and 1000 tons tension, on two column lines, each side of the main superstructure. In total there are 28 barrettes, 14 on each side of the building on a 48 × 12.8 m grid.

All barrettes are installed from the base of the excavation and vary from 12 to 20 m in depth. Because of the bending moments which act on these barrettes, they must be excavated, reinforced and concreted in a single operation, the largest concrete pour being in the order of 200 m³ allowing for overbreak.

It was vitally important that this work started on site as soon as the first area of excavation reached formation level. This was in October 1972. The contracts were sent out in July, the successful contractor being SEPICOS, the French affiliate of ICOS Ltd.

To give some idea of the organizational, design and planning problems, all of the earlier contracts were still in operation when SEPICOS started on site.

Fig. 5
Main service duct in cuvelage showing high quality of concrete finishes being achieved.
(Photo : Laurent Rousseau)

Conclusion

This short article does not give an opportunity to convey the extent of sophistication that has been achieved in this work in a very short space of time.

However, it does go some way to show the speed with which the French construction industry can operate. Arups have been able to match this speed of construction with speed of design without sacrificing any of the design quality.

Centre Beaubourg: The steelwork

Lennart Grut

From the beginning the steel structure has had 48 m between columns. The span has occasionally been larger, but it has never been less than 48 m. Six metres outside the principal columns are tension ties and in the space between is the main horizontal circulation. To span 48 m and carry exhibition and library loads requires a lot of steel. It has always been of great importance to the architect that the steel be exposed, so the quality and form have been an important design constraint. This constraint was rationalized into the theory that the lighter the steel the better. For a given strength, particularly in tension, there is nothing that appears lighter than a solid section. Thus the tension members became solid rounds and all the compression members circular tubes.

A typical cross-section has the two compression columns 48 m apart, flanked by the two tension columns a further 6 m away on either side. Between them and extending in until it hits the façade of the building, which is 1.6 m in from the compression column line, is a cast steel piece known as the gerberette (after Gerber, a German gentleman). Between the gerberettes and wholly inside the building are simply supported beams. The joints between the gerberettes and the column and the gerberettes and the beam are pinned. This frame, which occurs at 12.8 m intervals, is stabilized by the horizontal stiffness of the floors spanning between the stabilized frames at each end of the building. These stabilized frames are no more than a standard frame connected to act as a stiff plane. The floors, which act as the horizontal beams, are of composite construction. The floor is sub-divided into panels which are connected to provide rigidity but to avoid excessive stress from the action of the building in temperature and bending. Longitudinal stability is provided by the bracing in the tension plane. The stability of the columns is then treated for the building as a whole and the stability loads are transferred to the ends of the building and to the longitudinal bracing by the same route as the wind and temperature effects.

Because the steelwork is visible, the detailing expresses the action of the joint or element. Thus the pin connection between the gerberette and the column is a circular bearing. The gerberette is made in cast steel. This feature has been expanded to give continuity between detailing of the beam and the outside of the building. The use of cast steel provides a flexible material which can be adapted to the needs of the detailing and will hopefully not look too aggressive in use.

The choice of solid steel sections has a bonus when considering the fire resistance of the steel as the heat absorption qualities of the solid sections add to their resistance and reduce the amount of fire protection required. The method of fire protection for the internal steelwork has not yet been chosen, but will probably be either an intumescent paint capable of giving two hours protection, so far used only in the United States, or a multi-layer system based on the use of a stainless steel outer shield with different insulation layers. This has been developed by the AERE at Harwell. Both give a close fitting solution and add very little to the dimensions of the steelwork. It is hoped that the steel outside can be left untreated by the use of fire screens in the façade and by adapting the principle that a proportion of the members are redundant. The columns are waterfilled.

Problems have arisen with regard to the welding of the large steel sections, the specifving and welding to the castings, the manufacture and erection of the beams which, it is anticipated, will be erected in one piece, and difficulties have also been encountered with the specification of the steelwork to reduce likelihood of brittle fracture to an insignificant level. However, as the design is not yet finished, we cannot say how all these problems will be resolved, but we hope that in about 18 months time, 12,000 tons of steel will be standing erect in the centre of Paris.

Centre Beaubourg: The engineering services

Harvey Smith

Arups are responsible for all the services, including HVAC, electrical, plumbing and fire protection systems, lifts and escalators, but excluding the central control system, lighting and horizontal transport.

First impressions might well be of complex services installations, some 24 air conditioning plants ranging from single duct of cold fresh air to dual duct variable volume systems, 20 kV substations, extensive medium voltage and light current installations and plumbing systems for the provision of movable toilets. The range of activities in the Centre also gives an impression of complexity. We have to provide good standards of comfort and services for museums, libraries, a restaurant, theatre, conference rooms, storage, workshops, offices, a computer, car park and many other activities including a medical centre and television studios. In the event we flatter ourselves on having achieved very simple designs for all systems and, while this has required some compromise between the needs of different activities, it also contributes to the requirement of our brief that widely differing activities should be free to move from one part of the building to another.

In a building of this kind it is easy, too easy, to see difficulties arising from the wide range of specialist activities, the advanced architecture, the short construction programme and the strict financial control, to say nothing of working in strange surroundings, in a foreign language. In fact all these things tend to stimulate activity and to act as constraints which simplify selection and decision.

The total floor area of the building is about 80,000 m² and it will not quite fit into Trafalgar Square. The estimated weight of ducting is just under 1000 tons, about 8 km in the superstructure, ranging from 0.6 m to 1.2 m in diameter. We shall use several km of 20 kV cable, some 26 transformers rated at 1250 kVA and about 30 km of medium voltage cable for the main electrical distribution system. We aim to construct at the rate of about £4,000,000 worth of services per annum.

The design conditions selected for the air conditioning systems are:

External:		
Summer	32°CDB	21°CWB
Winter	−8°CDB	80% RH
Internal:		
Summer	25°CDB ± 1°C	50% RH ± 10%
Winter	20°CDB ± 1°C	50% RH ± 10%

Internal loads, lighting power, and people amount to about 50 watts per m².

The regulations of the Préfecture de Paris normally require public buildings to have 100% fresh air, but a waiver has been requested for Centre Beaubourg on the grounds of the building's function and the use of electricity for all services.

Two energy studies showed an 'all electric' system to be viable and the heating plant will consist of three electric boilers bearing a total rating of about 6700 kW and four storage vessels each of 115 m³. The latter will be charged on the low night tariff and the stored heat used during the peak tariff periods in the mornings and evenings. Thermal storage will not meet the full day's load, but is the most economical under the tariff offered by Electricité de France. The central refrigeration plant will consist of three machines, two rated at 3×10^6 Fq per hr (1000 ton) and one rated at 4×10^6 Fq per hr (1330 tons). Some heat recovery will be provided and the two smaller machines will be fitted with double bundle condensers.

The superstructure will be served by 13 air handling plants located on the roof. Each plant will serve 12.8 m vertical ducts on the exterior of the Raymond façade. The systems will be high velocity, dual duct, variable volume and include filters, air washers, heaters and variable pitch axial flow force handling about 80,000 m³/hr. Air conditioning systems in the substructure will include the following:

Storage at 21 m level

Low velocity terminal reheat

Workshop areas at 21 m level

Low velocity multizone

Cinema, threatre and conference area at 27 m level

Low velocity multizone 100% fresh air

Main entrance and exhibition areas at 27 m, 32 m and 35 m levels

High velocity dual duct with variable volume

Computer at 27 m level

Background provided by low velocity reheat system and 'in room' units in the computer rooms

Laboratories and printing at 27 m level

High velocity, dual duct constant volume, 100% fresh air.

Electrical systems

All energy requirements for the building are to be supplied by electricity and the connected load is of the order of 30 MVA. Electricity will be supplied by Electricité de France from the Paris 20 kV network.

Standby generating plant will be provided to maintain essential services and will consist of three dual generators, having a total capacity of about 3000 kVA. These generators will be connected to the 20 kV system through their own transformers.

The heating and air conditioning plants absorb a large part of the electrical load and transformers will accordingly be located adjacent to these plants. Six transformers servicing the lighting and power systems for the superstructure will be located on the exterior of the Rue Renard façade.

Flexibility in use is a main consideration in design and all the MV distribution systems are planned with this in mind. Lighting may range from 100 to 500 lux and much higher intensities may be demanded by particular exhibits. Lighting circuits are thus designed to meet this range. Other electrical systems include automatic telephones, fire detection and clocks. A system of trunking will be provided for these and other light current systems to be added later by the client.

Plumbing and fire protection

Fire protection will be provided by 10 hydrants in the street surrounding the Centre, dry risers in the substructure and superstructure. Fire hoses will be carried by storage and pressurizers. Sprinklers will be installed wherever the activities permit and CO^2 installations in other areas.

Plumbing installations will include drainage, storage and pumping to the domestic hot and cold water systems, sanitary units (mobile) and refuse packing and disposal.

Centre Beaubourg: Organization of the project

Brian Watt

The problem of communication is aggravated not only by the obvious language difficulties, but also by the need to exchange information extremely rapidly for a project as large and as complicated as Centre Beaubourg which has to be built on a very short time scale. The decision to have joint offices for the design team was made very early in the project. The offices for Piano+Rogers and Ove Arup & Partners are also shared by the management contractor who has a small group responsible for the programming aspects and includes his project manager.

Ove Arup & Partners are responsible for all engineering aspects of the project as well as the quantity surveying. The Ove Arup & Partners' team is sub-divided into a number of groups to deal with problems relating to the various professional disciplines. Each of these groups is headed by a group leader who is totally responsible for all work carried out within his section and he liaises directly with the architects, the client and outside bodies. The activities of the groups are co-ordinated by the project manager who is assisted by a co-ordinator.

The design of the HVAC and electrical systems is being carried out under our direction by Cabinet Trouvin, a firm of French consulting engineers. Ove Arup & Partners have the equivalent of a group leader for these two design areas also.

The client has a team of approximately 35 people including architects, planners, engineers and financial controllers. A so-called programmation team was set up before the competition stage to define the brief for the project and this group has continued to operate as intermediaries between the architects and the different future users of Centre Beaubourg. Meetings are held at frequent intervals between the design team, the programmation team of the EPCB and the users to ensure that the brief is being executed in accordance with their requirements.

In addition to the responsibility for establishing the site, the management contractor has a team of engineers responsible for the different building disciplines who work closely with the design team. These engineers become responsible for the site supervision when the pertinent sub-contractors commence work on site.

The design team's interests on site are looked after by our own site staff supplemented by back-up from our offices.

The project has been split into approximately 20 major contracts, the first of which was let in June 1972. By March 1973, five of these sub-contracts had been let and the sixth issued to tender. The client has a fairly rigorous check procedure for each contract and before the contract dossier is issued to prospective tenderers, it is submitted first to the client for his approval and secondly to a body called the Bureau de Marches which is responsible for approving all contracts issued for Centre Beaubourg. After the return of tenders, a report is submitted to the client with a recommendation as to whom it should be awarded. After approval by the client, this is passed to the Commission de Marches, a Government tender board which vets all major contracts for the State. As it is a management contract, the contract is actually signed between the client and the management contractor who then signs a sub-contract with the successful tenderer.

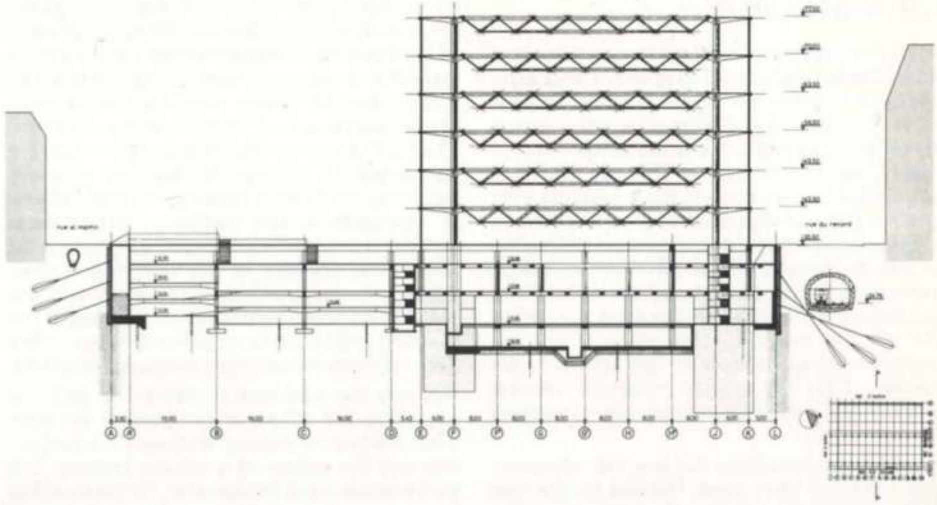
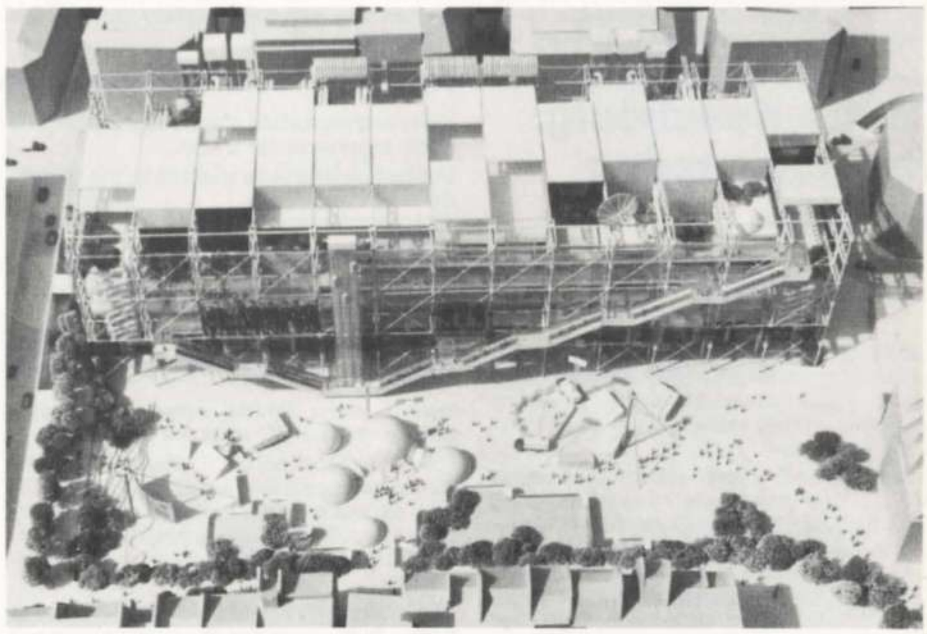
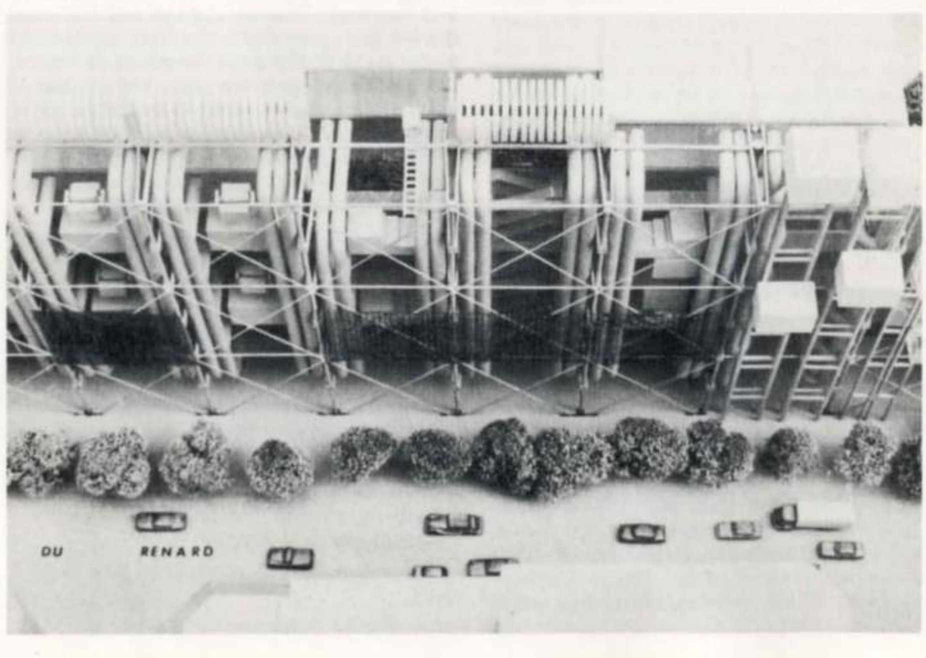


Fig. 1
Cross-section





Figs. 2 & 3
Model of final scheme

On the bending of architectural laminated glass

John Hooper

This paper appeared in the April 1973 issue of the International Journal of Mechanical Sciences.

Summary

As a result of its numerous environmental qualities, laminated safety glass is being used to an increasing extent in the field of architectural glazing. Its use in the manufacture of aircraft and automobile windscreens is well established, and the impact resistance of such laminates has been extensively studied. However, little work appears to have been done on the response of architectural laminated glass to normal structural loading. In this context, an architectural laminate is defined as comprising two glass layers of arbitrary thickness together with an adhesive plastic interlayer.

The aim of the present work is to provide an insight into the fundamental behaviour of architectural laminated glass in bending. To this end, theoretical and experimental studies have been made concerning the action of laminated glass beams in four-point bending. Closed-form expressions are derived for the interfacial shear traction and central deflection, and relevant numerical values are given. Experimental results are also presented; these relate to a series of tests on small laminated glass beams subjected to both transient and sustained loading at various ambient temperatures.

In general, the degree of coupling between the two glass layers is shown to be chiefly dependent upon the shear modulus of the interlayer, which in turn is found to be a function of both the ambient temperature and the duration of loading; in this connection, basic data are given on interlayer shear stiffness which can be utilized in subsequent structural analyses of architectural laminates.

Notation

W	total applied load
M	bending moment
T	interfacial shear force
q	interfacial shear stress
x, y	rectangular co-ordinates
B	beam width
A	cross-sectional area
I	second moment of area
l	distance between centroids of glass layers
t_i	layer thickness ($i=1, 2, 3$)
m	$(t_1^3+t_2^3)/t_1$
n	$(t_1^3+t_2^3)/t_2$
L	half-span
a, b	length
η	a/L
h	$a+L$
ξ	x/L
α, β, γ	parameters describing section properties
E_g	Young's modulus of glass
G_p	shear modulus of plastic interlayer
σ	bending stress
y	deflection
$H(x-c)$	Heaviside unit step function
s	Laplace transform parameter
K_{ij}	influence factor ($i, j=1, 2, 3$)
t	time
u, v, w, p	constants
$*$	superscript denoting maximum value

Introduction

In the broadest sense, glass laminates comprise alternate layers of glass and plastic which are joined together in adhesive contact. Such laminates have long been used in the manufacture of aircraft and automobile windscreens, but their use in architectural glazing is a fairly recent development. The number of individual layers which makes up the glass laminate differs widely according to application. In the case of aircraft windscreens, for example, there are commonly seven or more constituent layers, but in architectural applications, as in automobile glazing, laminates normally consist of only three layers. In this context, therefore, architectural laminated glass is considered to comprise two layers of plate or float glass, not necessarily of equal thickness, separated by an adhesive plastic interlayer.

In architectural applications, there are two principal advantages of laminated glass over monolithic plate or float glass. Firstly, the material properties of architectural laminates are such that when glass fracture occurs, the individual fragments remain adhered to the plastic interlayer and complete collapse of the glazed member is prevented. This capacity of a fractured glass laminate to remain substantially intact for a reasonable length of time classifies the laminate itself as a safety glass. Secondly, architectural laminates are constructed in such a way that they can be effectively employed in the control of shading and solar heat gain in buildings. This is normally achieved by applying a reflective metallic coating to one of the inner glass surfaces prior to laminating, although tinted glass or plastic layers can also be used to similar effect.

The principal disadvantage of architectural laminated glass, at least from the structural point of view, is its relatively low bending strength compared with monolithic glass of the same overall thickness. This reduced strength stems from the decrease in bending stiffness of the laminate due to the presence of the plastic interlayer, and it is the extent or degree of coupling between the two glass layers which is the subject of the present investigation.

In the design of aircraft and automobile windscreens, the primary structural requirement is one of adequate impact resistance, and this aspect has been the subject of extensive study within the industry. Architectural laminates, on the other hand, are primarily required to withstand bending moments induced by wind pressure, snow loading and self-weight acting over comparatively large spans, and yet very little work appears to have been carried out on the bending resistance of such laminates.

As a means of assessing the fundamental behaviour of architectural laminates in bending, attention is here directed towards a study of laminated beams subjected to four-point bending. Theoretical expressions are developed which account for the shear tractions at the glass–plastic interface; these in turn enable the glass layer bending stresses and central deflection to be determined. The theory then forms a basis for the interpretation of a series of bending tests on laminated glass beams of various cross-sectional proportions. This combined approach provides a useful insight into the bending mechanism of architectural laminates, and serves to produce the basic data on interlayer shear stiffness which are required in the analysis of laminated glass plates subjected to various loading and temperature conditions.

General theory

Procedure and assumptions

The method used here to solve the plane elastic problem of laminated glass in bending originates from a solution by Chitty[1] to a problem on the bending of parallel beams interconnected by cross-members. In this solution, the method of approach centred on replacing the discrete assemblage of interconnecting cross-members by a continuous medium of equivalent stiffness, the medium itself being firmly attached to the beams at each interface. But this latter condition corresponds precisely to that prevailing in the case of architectural laminated glass, in which a relatively soft continuous layer is confined between two glass layers, and remains in adhesive contact with them during bending. Thus the values of interfacial shear force determined in such an analysis can be directly applied to give the required values of bending stress in the two glass layers and also the shear stress transmitted by the plastic interlayer.

It is interesting to note that similar methods of analysis have been employed by Eriksson[2], Rosman[3] and others in connection with the structural analysis of plane coupled shear walls of a type often encountered in tall buildings. Furthermore, work of a similar nature is to be found in the early development of so-called sandwich beam theory; for example, that of van der Neut[4] on the bending of wooden box beams, and of Hoff and Mautner[5] on the bending of various sandwich-type elements used in aircraft construction.

The assumptions implicit in the following analysis are:

(a) The materials forming the laminate are isotropic and display linear elastic behaviour.

(*b*) Deflections are small, enabling the effects of membrane action to be neglected.

(c) No slip occurs at the interface between glass and plastic.

(d) The plastic interlayer itself offers no resistance to bending, other than by resisting relative displacement of the adjoining glass surfaces.

(e) Shear strains, other than those in the interlayer, are small and their contribution to the distortion of the glass layers is negligible.

(f) The thickness of the plastic layer remains constant, i.e. strains due to the induced normal stresses in the interlayer are neglected and the two glass layers follow the same deflection curve.

In practice, assumptions (d)–(f) are almost always valid, and the conditions in (b) and (c) are usually maintained at working loads, particularly for laminated glass of relatively thick overall section. Concerning assumption (a), both glass and plastic are isotropic and can reasonably be considered as linear elastic materials under transient loading; under long-term loading, the glass layers remain almost perfectly elastic but the plastic interlayer often undergoes substantial creep deformation. The implications of this non-linearity are discussed later, but firstly attention is given to the problem in which assumption (a) applies.

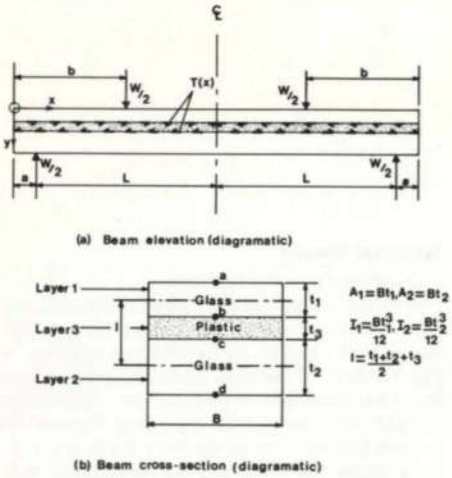


Fig. 1
Notation used in analysis of laminated beams

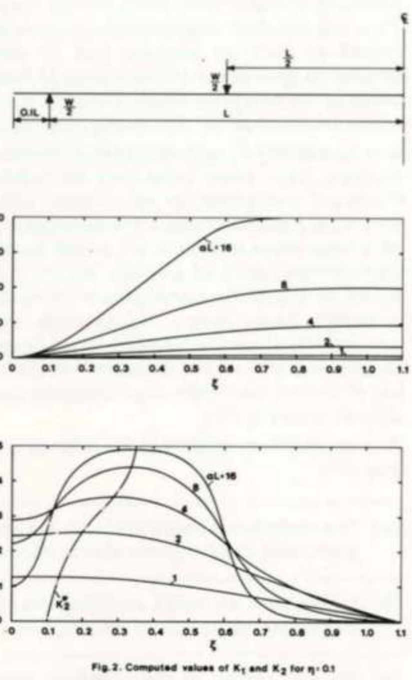


Fig. 2. Computed values of K_1 and K_2 for $\eta=0.1$

Fig. 2
Computed values of K_1 and K_2 for η=0.1.

Elastic solution for generalized four-point loading

If a beam comprising two outer layers of the same material, but not necessarily the same thickness, and a confined interlayer of a much softer material, is subjected to an applied bending moment, $M(x)$, then the governing differential equation for the interfacial shear force, $T(x)$, is:

$$\frac{d^2T}{dx^2}-\alpha^2T+\beta M=0, \tag{1}$$

where

$$\alpha^2=\beta\gamma l,\ \beta=\frac{G_p}{E_g}\frac{Bl}{It_3},\ \gamma=1+\frac{AI}{A_1A_2l^2}. \tag{2}$$

In addition, G_p denotes the shear modulus of the plastic interlayer, E_g denotes Young's modulus of the glass, $A=A_1+A_2$, $I=I_1+I_2$ and the remaining notation is given in Fig. 1. In this context, the interfacial shear force, $T(x)$, can also be interpreted as the resultant longitudinal load in one of the glass layers.

The particular problem to be solved is that of determining the distribution of interfacial shear force along the length of a simply supported beam subjected to four-point loading, as indicated in Fig. 1 (a). A total applied load of W acts upon a beam having a clear span $2L$ and a cantilever section of length a beyond each support. As the loading is symmetric about the beam centre, the bending moment is of constant magnitude over a central length L, and values of interfacial shear force need only be determined for one half of the beam. It may be noted, however, that although this shear force must be single-valued at each of the two loading points, three different functions are required to define the distribution of interfacial shear force along the length of the half-beam. Considering the left-hand side of the beam shown in Fig. 1 (a), it is initially convenient to let the downward applied load to be positioned at the point $x=b$, and also to let $h=a+L$. Now the bending moment in the region $0\leqslant x\leqslant h$ can be expressed in the operational form

$$M(x)=\frac{W}{2}[(x-a)H(x-a)-(x-b)H(x-b)], \tag{3}$$

where Heaviside's unit step function is defined as

$$H(x-c)=\begin{cases}0, & x<c,\\ 1, & x>c.\end{cases}$$

Substituting (3) into (1) and taking the Laplace transform of the latter gives

$$s^2t(s)-sT(0)-T'(0)-\alpha^2t(s)+\frac{\beta W}{2s^2}[\exp(-as)-\exp(-bs)]=0, \tag{4}$$

where s denotes the transform parameter. With the boundary condition $T(0)=0$, (4) may be inverted to give

$$T(x)=T'(0)\frac{\sinh\alpha x}{\alpha}-\frac{\beta W}{\alpha^3}\left\{\sinh[\alpha(x-a)]-\sinh[\alpha(x-b)]-\alpha(b-a)\right\}. \tag{5}$$

The remaining unknown, $T'(0)$, is readily determined from the second boundary condition, $T'(x)=0$ at $x=h$. The required expressions for interfacial shear force in each of the three zones are therefore as follows:

For $0\leqslant x\leqslant a$,

$$T(x)=\frac{\beta W}{2\alpha^3}\left(\frac{\cosh[\alpha(h-a)]-\cosh[\alpha(h-b)]}{\cosh\alpha h}\right)\sinh\alpha x. \tag{6}$$

For $a\leqslant x\leqslant b$,

$$T(x)=\frac{\beta W}{2\alpha^3}\left[\left(\frac{\cosh[\alpha(h-a)]-\cosh[\alpha(h-b)]}{\cosh\alpha h}\right)\sinh\alpha x-\sinh[\alpha(x-a)]+\alpha(x-a)\right]. \tag{7}$$

For $b\leqslant x\leqslant h$,

$$T(x)=\frac{\beta W}{2\alpha^3}\left[\left(\frac{\cosh[\alpha(h-a)]-\cosh[\alpha(h-b)]}{\cosh\alpha h}\right)\sinh\alpha x-\sinh[\alpha(x-a)]+\sinh[(\alpha x-b)]+\alpha(b-a)\right] \tag{8}$$

Corresponding expressions for the shear stress, $q(x)$, transmitted by the interlayer, can be directly obtained from

$$q(x)=dT/dx. \tag{9}$$

Thus for any particular loading case, the principal variables may be written as

$$T(x)=\frac{\beta W}{\alpha^3}K_{1j},\ q(x)=\frac{\beta W}{B\alpha^2}K_{2j}, \tag{10}$$

where K_1, K_2 are influence factors and $j=1, 2, 3$ corresponding to the three loading sections of the beam.

Elastic solution for standard four-point loading

With the additional notation

$$\xi=x/L,\ \eta=a/L,\ A=\frac{\cosh\alpha L-\cosh(\alpha L/2)}{\cosh[\alpha L(1+\eta)]}, \tag{11}$$

the required equations for the particular case when $b=a+L/2$, normally referred to as standard four-point loading, are as given below:

For $0\leqslant x\leqslant a$,

$$K_{11}=\frac{A}{2}\sinh\ \xi\alpha L, \tag{12}$$

$$K_{21}=\frac{A}{2}\cosh\ \xi\alpha L. \tag{13}$$

For $a\leqslant x\leqslant (a+L/2)$,

$$K_{12}=\tfrac{1}{2}\{A\sinh\ \xi\alpha L-\sinh[\alpha L(\xi-\eta)]+\alpha L(\xi-\eta)\}, \tag{14}$$

$$K_{22}=\tfrac{1}{2}\{A\cosh\ \xi\alpha L-\cosh[\alpha L(\xi-\eta)]+1\}. \tag{15}$$

For $(a+L/2)\leqslant x\leqslant (a+L)$,

$$K_{13}=\tfrac{1}{2}\{A\sinh\ \xi\alpha L-\sinh[\alpha L(\xi-\eta)]+\sinh[\alpha L(\xi-\eta-\tfrac{1}{2})+\tfrac{1}{2}\alpha L\}, \tag{16}$$

$$K_{23}=\tfrac{1}{2}\{A\cosh\ \xi\alpha L-\cosh[\alpha L(\xi-\eta)]+\cosh[\alpha L(\xi-\eta-\tfrac{1}{2})]\}. \tag{17}$$

Values of the factors K_1, K_2 are plotted in Fig. 2 for $\eta=0{\cdot}1$. The maximum value of K_1, denoted by K_1^*, always occurs at the beam centre; the maximum value of K_2, namely K_2^*, always occurs in the zone $a \leqslant x \leqslant (a+L/2)$, but its precise position depends upon αL and η. By differentiation of (15), the required position is given by

$$\xi=\frac{1}{2\alpha L}\log_e\left(\frac{\cosh \alpha L-\cosh (\alpha L/2)-\exp (\eta\alpha L) \cosh [\alpha L(1+\eta)]}{\cosh \alpha L-\cosh (\alpha L/2)-\exp (-\eta\alpha L) \cosh [\alpha L(1+\eta)]}\right) \tag{18}$$

and the locus of K_2^* for $\eta=0.1$ is shown by the chain-dotted line in Fig. 2.

With the interfacial shear forces now known, the maximum bending stresses, σ^*, in the two glass layers can be calculated. For layer 1,

$$\sigma_{a,b}^*=\pm\frac{t_1}{2I}(M-Tl)+\frac{T}{Bt_1}, \tag{19}$$

where the subscripts a, b refer to the positions indicated in Fig. 1(b). A similar expression holds for layer 2, and in each case $M=WL/4$. With γ as defined in (2), and denoting $m=(t_1^3+t_2^3)/t_1$, $n=(t^3+t^3)/t_2$, the resulting stress equations are

$$\sigma_a^*=\frac{WL}{mB}\left[\frac{3}{2}+\frac{K_1^*}{\gamma\alpha L}\left(\frac{m}{lt_1}-6\right)\right], \tag{20}$$

$$\sigma_b^*=\frac{WL}{mB}\left[\frac{K_1^*}{\gamma\alpha L}\left(\frac{m}{lt_1}+6\right)-\frac{3}{2}\right], \tag{21}$$

$$\sigma_c^*=\frac{WL}{nB}\left[\frac{3}{2}-\frac{K_1^*}{\gamma\alpha L}\left(\frac{n}{lt_2}+6\right)\right], \tag{22}$$

$$\sigma_d^*=\frac{WL}{nB}\left[\frac{K_1^*}{\gamma\alpha L}\left(6-\frac{n}{lt_2}\right)-\frac{3}{2}\right], \tag{23}$$

where tensile stresses are taken as negative.

The deflection of the beam centre, y^*, relative to the two supports may be found by integration of the equation

$$E_g I\frac{d^2 y}{dx^2}=-(M-Tl), \tag{24}$$

the boundary conditions being $y=0$ at $x=a$, and $y'=0$ at $x=(a+L)$. The result is

$$y^*=\frac{11WL^3}{96E_g I}K_3, \tag{25}$$

where

$$K_3=1-\frac{1}{\gamma}-\frac{48}{11\gamma(\alpha L)^3}\left[\left(\frac{\cosh \alpha L-1}{\cosh[\alpha L(1+\eta)]}\right)\{\sinh \eta\alpha L-\sinh[\alpha L(1+\eta)]\}\right.$$
$$-\left(\frac{\cosh(\alpha L/2)-1}{\cosh[\alpha L(1+\eta)]}\right)\{\sinh[\alpha L(\tfrac{1}{2}+\eta)]-\sinh[\alpha L(1+\eta)]$$
$$\left.-\frac{\alpha L}{2}\cosh[\alpha L(\tfrac{1}{2}+\eta)]\}+\sinh \alpha L-\sinh\frac{\alpha L}{2}-\frac{\alpha L}{2}\right]. \tag{26}$$

Values of K_3 for $\eta=0{\cdot}1$ are plotted in Fig. 3. Clearly the deflection is markedly sensitive to the modulus of the interlayer for the lower values of αL, whereas for $\alpha L>8$, K_3 is nearly equal to the limiting value of $(1-1/\gamma)$.

Finally, it is useful to examine the extent to which the results are affected by the length of the cantilever section of the laminated beam. The effect is shown in Fig. 4, which gives results for $\alpha L=1{\cdot}0$ (soft interlayer) and $\alpha L=4{\cdot}0$ (hard interlayer) as η varies from 0 to 0·5. It is evident from Fig. 4(a) that as η increases, the maximum interfacial shear force also increases, and is accompanied by a corresponding decrease in the maximum shear stress transmitted by the interlayer. Likewise, Fig. 4(b) shows the manner in which the deflection decreases as η increases, and includes the effect of the parameter γ. In general, therefore, the length of the cantilever section has a significant effect upon the behaviour of laminated beams in four-point bending, and should thus be taken into account in the analysis of test results. Where possible, the cantilever length should be standardized, preferably at $\eta=0{\cdot}1$.

Non-elastic effects

Under conditions of short-term loading, it is reasonable to consider the plastic interlayer as a linear elastic material, but for long-term loading this assertion is no longer valid. In this latter case, the glass layers will remain almost perfectly elastic, but the shear tractions acting on the interlayer will generally give rise to substantial creep deformations in the plastic, and thus modify the bending stresses in the glass layers.

An assessment of the continuous variation of T, q, y^* with time could be made by replacing the elastic shear modulus, G_p, by $G_p(t)$, the variation of shear modulus with time, and treating the problem within the framework of linear viscoelastic theory. In this approach, the function $G_p(t)$ could be deduced experimentally at a given ambient temperature by observing the shear deformation with time, $\delta(t)$, of laminated glass specimens subjected to constant shear traction. However, this shear deformation for the type of plastic used as the interlayer material would probably take the form

$$\delta(t)=u+v[1-\exp(-pt)]+wt, \tag{27}$$

where u, v, w, p are constants and t is the time variable.

In most practical situations involving the sustained loading of architectural laminates, creep deformation of the interlayer will take place comparatively quickly, and so it is the limiting value of $\delta(t)$ which is of predominant interest. It is evident from (27) that as $t\rightarrow\infty$, $G_p(t)\rightarrow 0$, which implies that for long-term static loading, glass bending stresses may be assessed on the assumption that the laminate consists of two independent glass layers separated by a constant distance, with no coupling effect due to the interlayer.

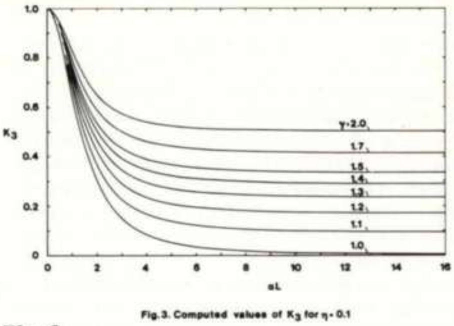


Fig. 3
Computed values of K_3 for $\eta=0{\cdot}1$.

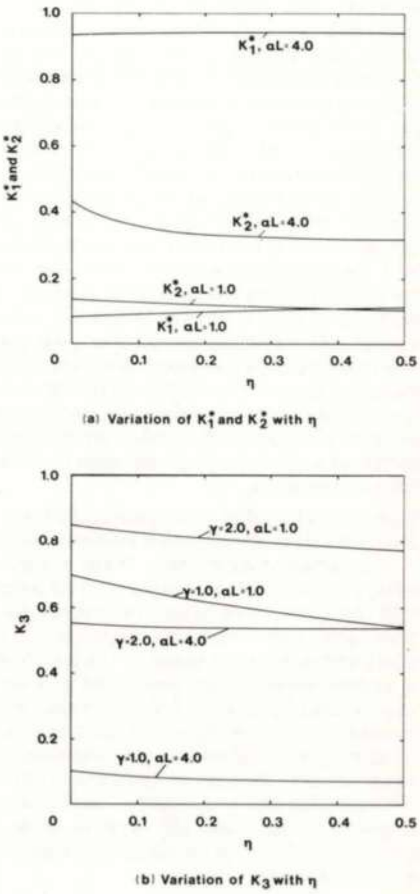


Fig. 4
Effect of η on analytical results for laminated beams

Experiments and discussion of results

Scope of experiments

Two types of experiment were carried out, each involving the loading of laminated beams in standard four-point bending. Firstly, a number of small strain-gauged beams were loaded fairly rapidly by means of a universal testing machine; deflections were measured at the centre of the beams, and strain gauge measurements enabled the bending stresses across the laminated section to be determined. Secondly, tests were carried out in which a number of beams were subjected to sustained load at various ambient temperatures, the central deflection of each beam being measured at intervals throughout the test period.

All beams tested were approximately 559 mm long and 51 mm wide; the clear span between supports was 508 mm, giving $\eta=0{\cdot}1$. The applied loads were symmetrical about the beam centre, and spaced 254 mm apart. The glass used was clear float or plate, with the exception of one set of creep specimens in which one of the constituent layers was tinted plate glass. The plastic interlayer consisted of polyvinyl butyral (abbreviated to PVB) resin containing various proportions of a given plasticizer, namely triethylene glycol di (2-ethylbutyrate). This interlayer is normally produced in the form of a flexible opaque sheet which becomes transparent during the laminating process. The moisture content of the interlayer, which mainly controls the degree of adhesion to glass, was measured in a number of laminated specimens by means of near-infra-red spectroscopy. In each case the moisture content was found to be approximately 0.4%, a value which corresponds to a relatively high adhesion between the two materials.

Most of the bending tests were carried out on laminated glass beams which incorporated one of two different interlayers; these contained either 21 or 41 parts by weight per 100 parts of PVB resin, and have been designated here as 'hard' and 'soft' interlayers respectively. The hard interlayer is used mainly in the production of aircraft windscreens, whilst the soft interlayer is used for architectural laminated glass. In addition, one test was carried out on a beam containing a so-called 'impact' interlayer, this being almost identical in composition to the soft interlayer but formulated in such a way as to give a rather lower adhesion to glass. Its principal use is in the manufacture of automobile windscreens.

Experiments on strain-gauged beams

Resistance foil strain gauges were affixed to *both* inner and outer glass surfaces of nine laminated glass beams of various cross-section. This was achieved by bonding gauges with thin lead wires onto one face of each glass layer prior to laminating. Particular care was taken in the bonding of the gauges; the gauge cement was oven-cured at a temperature well above that encountered in the subsequent laminating process, and then allowed to cool slowly to room temperature.

The individual glass beams were then laminated by the normal manufacturing process. The plastic interlayer was placed between the two glass layers, each with the gauged surface inwards. This initially loose combination was passed through a series of heated rollers to secure partial adhesion, and lamination finally completed by means of an autoclave. At this stage, strain gauges were bonded to the outer glass surfaces at the centre of the laminated beam.

The beams were loaded at an ambient temperature of 21 °C by means of a carefully calibrated universal testing machine. To ensure accurate alignment, loads were applied via a spherical seating to a steel bar incorporating two tilting rollers set 254 mm apart. Strain gauge and dial gauge readings were taken at several load increments, and each test took about three minutes to complete. In the present context, this is referred to as 'short-term' loading. For beams containing a soft interlayer, creep deformation was negligible and the observed load/deflection curves were sensibly linear. Some creep deformation did occur in beams with a hard interlayer, resulting in slightly non-linear load/deformation and strain/deformation curves; in these cases, deduced values of αL and G_p were based upon the initial tangent to the curves. Those beams of asymmetrical cross-section were loaded with the thinner glass layer in both upper and lower positions but, as expected, the measured deflections and surface strains were almost identical in magnitude for either of the two positions.

Table 1 : Values of αL and G_p deduced from measured strains; short-term loading of laminated glass beams at 21 °C

Nominal layer thickness (mm)					
t_1	t_2	t_3	Interlayer type	αL	G_p (*MPa*)
8	8	0·76	Hard	3·7	9·2
6	10	0·76	Hard	4·0	12·6
3	12	0·76	Hard	3·2	10·2
6	8	0·76	Hard	3·8	8·9
6	6	0·76	Hard	4·8	10·5
6	10	0·38	Hard	6·0	15·2
6	10	1·02	Hard	3·4	11·7
6	10	0·76	Soft	1·0	0·8
6	10	0·76	Impact	1·0	0·8

As a means of interpreting the experimental data, values of K_3 were computed using (25) in conjunction with the measured load/deflection curves, and the corresponding value of αL read off from Fig. 3. A second set of αL values was then obtained by comparing the measured strains at the beam centre with those calculated on the basis of (20)–(23). For beams having a soft or impact interlayer, the two methods yielded almost identical values of αL. For beams with a hard interlayer, the αL values deduced on the basis of measured deflections were somewhat erratic; reference to Fig. 3 shows that this might be expected, as K_3 is distinctly insensitive to αL for relatively stiff interlayers. Strain-gauge measurements, on the other hand, gave more consistent results; there was good agreement between measured and computed strains in all cases, and values of αL deduced by this method are listed in Table 1. Also listed are the corresponding values of G_p, calculated using (2) in conjunction with a Young's modulus for glass of 72·4 *GPa* derived from bending tests on monolithic beams.

It is immediately apparent from Table 1 that, at an ambient temperature of 21 °C, a reduction in the proportion of plasticizer from 41 to 21 parts gives rise to an increase in interlayer shear modulus of approximately one order of magnitude. Considering 0·76 mm thick interlayers, for example, the average value of G_p is 10·3 *MPa* for the hard material, compared with a value of 0·8 *MPa* for the soft material. For thicker interlayers, the results for laminates comprising 6 mm and 10 mm glass layers and a hard interlayer indicate that the value of G_p remains approximately the same, but as the thickness decreases, the value of G_p apparently increases. In this connection, the estimated shear modulus for a 0·38 mm thick hard interlayer is 15·2 *MPa*, some 50% higher than the corresponding value for a 0·76 mm thick interlayer. A similar trend has been observed by Quenett[6], and may be due in part to the higher rate of strain to which the thinner interlayers are subjected. It may also be partly due to some additional confining or restraining effect at the plastic/glass interface, possibly in the nature of some 'boundary layer' phenomenon, which manifests itself only when the interlayer is comparatively thin.

Differences in shear moduli of the magnitude encountered in Table 1 for soft and hard interlayers are sufficient to induce widely differing bending stresses in the glass layers under conditions of short-term loading. This may be illustrated by considering the bending stresses at the centre of those test beams comprising nominal 6 mm and 10 mm glass layers and a 0·76 mm thick plastic interlayer. Actual beam dimensions were $B=50{\cdot}5$ mm, $t_1=5{\cdot}87$ mm, $t_2=9{\cdot}52$ mm, $t_3=0{\cdot}76$ mm, giving $l=8{\cdot}46$ mm and $\gamma=1{\cdot}343$. The applied load was 400 *N*, and stress values relate to the case where the beam is positioned with its thinner layer uppermost, tensile stresses being taken as negative. Fig. 5 shows the theoretical distribution of bending stress across this particular laminated section for both soft and hard interlayers with the values of αL corresponding to those given in Table 1.

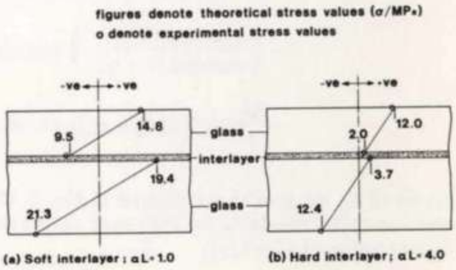


Fig. 5
Theoretical and experimental values of bending stress in laminated glass test beams with soft and hard PVB interlayers ($W=400\,N$, $L=254$ mm, $B=50.5$ mm, $t_1=5{\cdot}87$ mm, $t_2=9{\cdot}52$ mm, $t_3=0{\cdot}76$ mm).

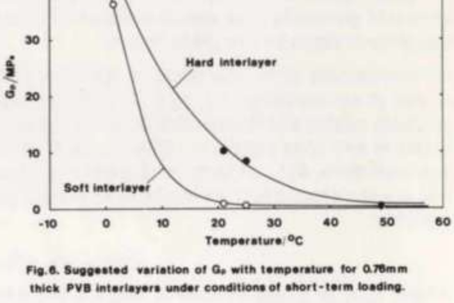


Fig.6. Suggested variation of G_p with temperature for 0.76mm thick PVB interlayers under conditions of short-term loading.

Fig. 6
Suggested variation of G_p with temperature for 0·76 mm thick PVB interlayers under conditions of short-term loading.

Also plotted are stress values derived from surface strain measurements, and these agree well with the corresponding theoretical results. Of particular significance in this example is that the maximum tensile bending stress within the laminate increases by 70% as αL is reduced from 4·0 to 1·0.

Creep experiments

In addition to the short-term loading tests described in the preceding section, a number of small laminated glass beams were subjected to sustained loading at various ambient temperatures. The beam dimensions and loading spans were the same as for the short-term tests, and the applied loads were in the form of dead-weights attached to hangers located at the quarter points. The duration of loading was approximately 80 days, and measurements of central deflection and ambient temperature were made at intervals throughout this period.

Each of the three creep-loading frames accommodated six laminated glass beams of various cross-sectional geometry, together with one monolithic glass beam which served as a control specimen. The three sets of identical beams were tested in temperature-controlled rooms.

The test temperatures were 1·4, 25 and 49°C, and the observed limits during the test period were ±0·4, ±0·2 and ±1·2°C, respectively.

At the start of the creep tests, the dead weights were applied fairly rapidly, with the beams fully loaded after about one minute. As for the previous tests on strain-gauged beams, this is arbitrarily considered as 'short-term' loading. The resulting load/deflection curves were linear in all cases, and values of αL and G_p, deduced from these curves by means of (25) and Fig. 3, are listed in Table 2. In addition, average values of G_p given in Tables 1 and 2 for 0·76 mm thick interlayers are plotted in Fig. 6, which also gives curves suggesting the continuous variation of G_p with temperature.

Reference to Fig. 6 clearly demonstrates the dominating influence of temperature upon the shear stiffness of the interlayer, with values of G_p differing by two orders of magnitude over a reasonably narrow temperature range. Hence it follows that where laminated glass is subjected to short-term loading, the stress distribution across the composite section will be strongly dependent upon the ambient temperature. This is emphasized by the stress values listed in Table 3 for those test beams comprising nominal 6 mm and 10 mm glass layers and a 0·76 mm thick interlayer. The values given are the maximum interfacial shear stress, q^*, computed from (10), (15) and (18), together with the maximum bending stress, σ_d^*, computed from (23). Clearly the effect of a rise in temperature is to reduce q^* and increase σ_d^*; in this particular example, the value of σ_d^* is approximately doubled over the temperature range of the tests. However, even at the lower temperatures, q^* is low compared to the anticipated shear strength of the glass/plastic bond, and this is reflected in the linearity of the measured load/deflection curves.

The combined effect of temperature and sustained load on the deflection characteristics of the laminated glass test beams is typified by the measured values given in Fig. 7 for three pairs of beams comprising nominal 6 mm and 10 mm glass layers and a 0·76 mm thick PVB interlayer. Once again, the results are seen to be markedly dependent on temperature. At 1·4°C, the initial central deflection of both beams was approximately 0·4 mm. This deflection did not increase with time for the beam with a hard interlayer, whereas the deflection of the beam containing a soft interlayer increased to almost three times its initial value. At 25°C, the initial deflections differed by a factor slightly greater than 2, but substantial creep of the hard interlayer resulted in the deflections of both beams converging with respect to each other to some common value in the region of 1·3 mm. At 49°C, the deflections of the two beams were similar throughout the test period; the creep component was small due to the low shear modulus of both types of interlayer at this temperature, the initial and final deflections being approximately 1·2 mm and 1·4 mm, respectively. An additional feature common to tests at all three temperatures was that the immediate upward deflection which occured on removal of the load closely matched the initial downward deflection measured at the beginning of each test.

As expected, the deflection of the monolithic glass beams remained almost constant throughout the test period, thereby suggesting that reliable upper and lower bounds for the central deflection of the laminated beams may be determined using (25) in conjunction with the limiting values of K_3 from (26). For zero interlayer shear stiffness, K_3 equals unity and $y^* =$ 1·41 mm; for interlayers with very high shear stiffness, $K_3 \simeq (1-1/\gamma)$ and $y^* = 0.36$ mm. Reference to Fig. 7 shows this latter value to be slightly less than the initial deflection of the beams at 1·4°C, whereas the former upper bound closely resembles the final measured deflection of the test beams at 49°C.

In conclusion, the results of the creep loading tests indicate that, except at comparatively low temperatures, the long-term response of architectural laminates subjected to sustained loading will be substantially the same for all types of PVB interlayers in common use. Unlimited creep deformation of the interlayer appears to take place in the manner suggested by (27) irrespective of the plasticizer content, so that after a sufficient length of time the shear modulus of the interlayer effectively reduces to zero. Under these circumstances, it is reasonable to compute bending stresses and deflections due to sustained loading on the assumption of two independent glass layers. If, however, a short-term load is subsequently applied to the already loaded section, the additional bending stresses can be estimated using an appropriate interlayer shear modulus based on the data given in Fig. 6. Following this approach, the stress field resulting from a combination of sustained and transient loading may be readily determined by superposition.

Table 2: Values of αL and G_p deduced from measured deflections of laminated glass beams subjected to short-term loading at different ambient temperatures

Nominal layer thickness (mm)			Interlayer type	αL			G_p (*MPa*)			
t_1	t_2	t_3		1·4°C	25.0°C	49.0°C	1.4°C	25.0°C	49.0°C	Remarks
8	8	0·76	Hard	8·5	2·9	0·83	48·9	5·7	0·45	
6	10	0·76	Hard	6·5	3·5	0·77	35·2	10·2	0·50	6 mm layer top
6	10	0·76	Hard	7·3	3·4	0·73	44·3	9·6	0·45	6 mm layer bottom
							42·8	8·5	0·45	Average G_p value
8	8	0·76	Soft	8·0	0·95	0·72	43·3	0·60	0·35	
6	10	0·76	Soft	6·5	0·87	0·71	35·2	0·65	0·40	6 mm layer top
6	10	0·76	Soft	6·1	0·78	0·67	31·0	0·50	0·35	6 mm tinted layer bottom
							36.5	0.60	0.40	Average G_p value

Table 3: Initial values of q^* and σ_d^* for creep test specimens ($W=285.1N$, $L=254$ mm, $B=51.6$ mm, $t_1=6.17$ mm, $t_2=9.96$ mm, $t_3=0.76$ mm)

Temperature (°C)	Interlayer type	Average αL	q^* (*MPa*)	σ^*_d (*MPa*)
1·4	Hard	6·9	0·20	7·4
25·0	Hard	3·4	0·16	8·3
49·0	Hard	0·75	0·04	14·6
1·4	Soft	6·3	0·19	7·5
25·0	Soft	0·82	0·04	14·3
49·0	Soft	0·69	0·03	15·1

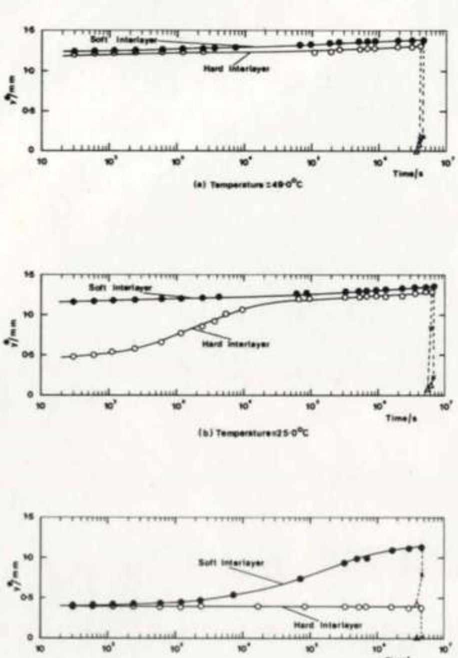


Fig. 7
Experimental creep curves for similar laminated glass beams loaded in standard four-point bending at various ambient temperatures ($W=285{\cdot}1\,N$, $L=254$ mm, $B=51{\cdot}6\pm1{\cdot}0$ mm, $t_1=6{\cdot}17\pm0{\cdot}10$ mm, $t_2=9{\cdot}96\pm0{\cdot}02$ mm, $t_3=0{\cdot}76$ mm; points X relate to immediate unloading, points Δ relate to seven days after unloading)

Conclusions

The present investigation demonstrates that the bending resistance of architectural laminated glass is principally dependent upon the thickness and shear modulus of the interlayer. However, those interlayers in common usage are thermoplastic materials whose physical properties, even under normal operating conditions, are found to be dependent upon plasticizer content, ambient temperature and duration of loading.

The load/deflection response of architectural laminates is sensibly linear for short-term loading, and the indications are that no interfacial slip occurs between the glass and plastic, at least at working loads. If the applied loads are sustained, then creep deformation takes place within the plastic interlayer and, except at relatively low temperatures, allows the glass layers to deflect as though they were separated at a constant distance by a material of zero shear modulus. But if a laminate undergoing sustained loading is subjected to additional transient loads, it will respond as a composite member having an interlayer shear modulus appropriate to its temperature.

For the purposes of structural design, architectural laminates which are likely to be subjected to sustained loads, e.g. snow or self-weight loading, should be considered as two independent glass layers, with no coupling effect due to the interlayer. For short-term loading, e.g. wind loading, glass bending stresses may be estimated on the basis of an interlayer shear modulus corresponding to the maximum temperature at which such loading is likely to occur, remembering that solar radiation may well raise the temperature of glazed laminates to well above that of the surrounding atmosphere. Where laminates are to be subjected to both sustained and transient loading, the combined values of bending stress in the glass layers may be obtained by superposition.

Acknowledgements

This work is the outcome of studies made in connection with the structural design of the glass walls for the Sydney Opera House in Australia. The experimental programme was financed by the Government of New South Wales. The short-term loading tests were carried out at the materials testing laboratory of the Public Works Department, Sydney. The sustained-load tests were carried out at the CSIRO Division of Food Preservation, Sydney. The laminated test specimens were supplied by Pilkington Bros. (Aust.) Pty. Ltd. of Geelong, Victoria, and by the French companies Boussois Souchon Neuvesel/Société Industrielle Triplex. The generous assistance of the above organisations, and also of John Blanchard, is gratefully acknowledged.

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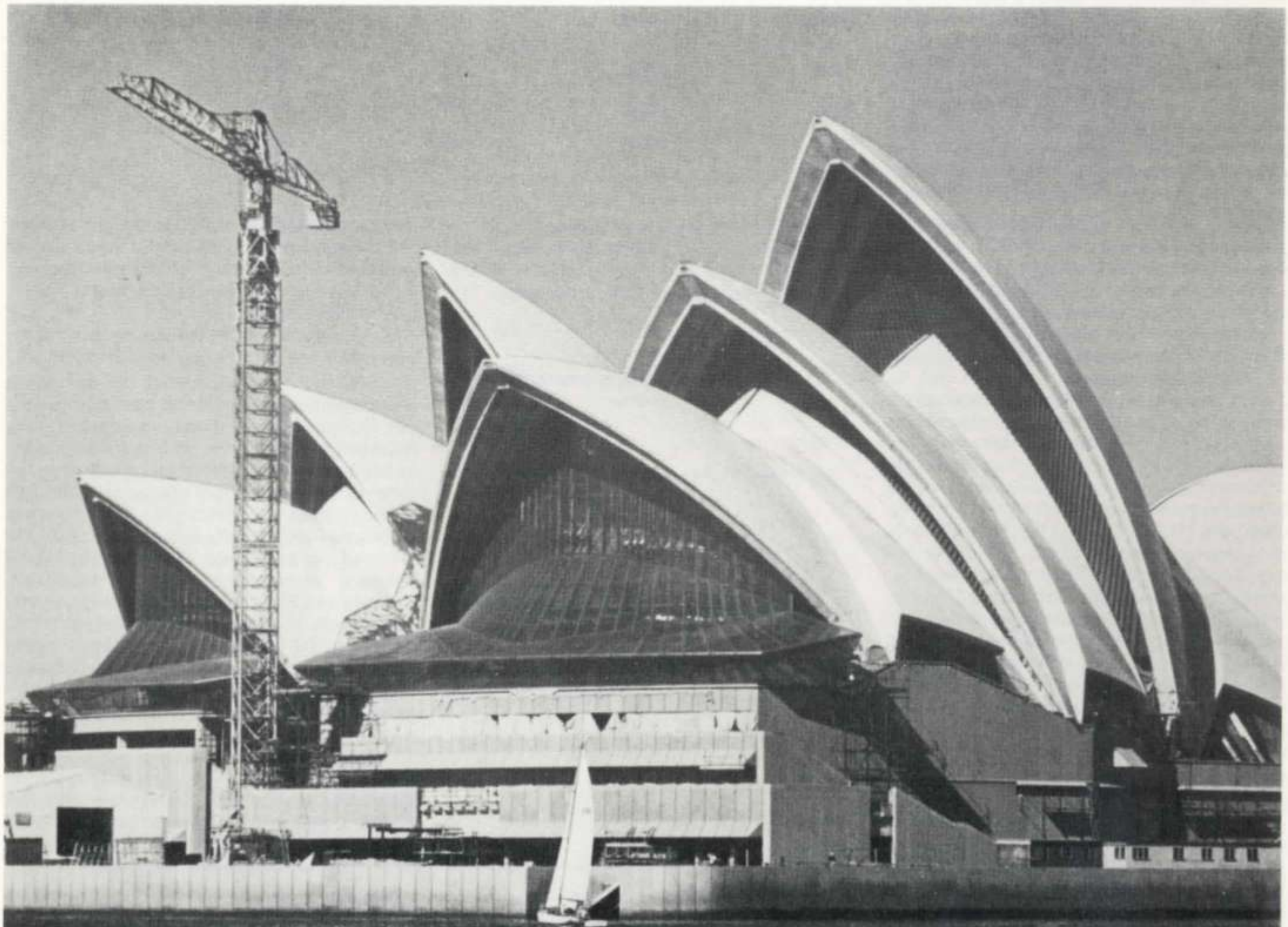
Fig. 8
Laminated glass walls on Sydney Opera House (Photo: Harry Sowden)



Editor's note: The next *Arup Journal* will be a Sydney Opera House special issue and will appear in October 1973 to coincide with the opening of the Opera House.

Examples on the use of desk-top computers for the solution of structural engineering problems

Wael Hussein

The Scottish Office took delivery of a Hewlett-Packard 9100B Programmable Calculator, in March 1970, and in common with the rest of the Partnership, attempted to make maximum use of its potential.

After the early 'honeymoon' stage, when many people were interested in carrying out programming work, it became apparent that the major contribution it had to make was as a production tool. To this end, considerable thought was given to enable it to achieve maximum effect and after a number of trial runs of alternatives, the format for program and data presentation as illustrated in the following paper was evolved. We have found that this format allows data preparation to be carried out without the machine and if necessary, the machine itself to be operated by non-technical staff. This latter point was felt to be useful in allowing a smaller office, which did not itself possess a machine, to make use of that in another office when speed was not absolutely essential.

Hamish Stears

Introduction

This paper originally appeared in the June 1973 issue of *Building Science* and is reproduced here with their permission. It describes the use of desk-top computers in the structural engineering practice. It is aimed at illuminating the potential of these machines in (a) providing an efficient and speedy design tool for medium size problems; and (b) standardizing the methods of analysis in design offices.

It is shown that desk-top computers are capable of furnishing complete design information, e.g. member sizes, steel reinforcement, etc., from the basic knowledge of the structure's geometry and loading conditions. This will reduce the work load generated by day to day standard problems thus saving time for more imaginative thinking and examining other alternatives.

Examples of programs written for the analysis and design of a number of structural elements are reported to illustrate the advantages of these calculators.

Desk-top computers are electronic calculators with memory store. These machines can be used as desk calculators capable of performing the usual arithmetical operations such as addition, subtraction, etc., and functional operations such as $\sqrt{x}$, log*x*, sin*x*, etc., at a single keystroke. They can also be programmed to carry out a sequence of instructions on a given set of data which may involve storage, automatic printing and plotting of the results.

These machines provide an automatic aid for solving medium-size problems, which are too small for digital computers, yet too tedious and repetitive for hand calculation.

The programming of desk-top calculators is generally simple and does not require knowledge of high-level digital computer languages. The programs may incorporate self-checking operations, e.g. $\Sigma X = \Sigma Y = \Sigma M = O$, and diagnostic codes such as a red light signal or a series of numbers to indicate illegal or illogical operations.

The programs may be recorded on magnetic cards or tapes which can be protected against accidental damage.

Input data are usually kept to a minimum, e.g. the structural properties and loading conditions, and hence no hand calculations are required prior to using a program.

Experience has shown that the major source of error when using these machines is wrong input data. This can be minimized by checking the print-out of the input information before the execution of the computation and by including a branching facility to any part of the program in order to correct wrong entries.

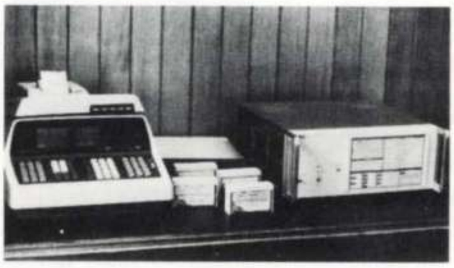


Fig. 1
The Hewlett-Packard system

A user's manual which incorporates a description of the programs, their limitations specific instructions on how to use them, and perhaps illustrated examples, will guide the users in operating the programs and satisfy their curiosity on how the answer is worked out.

It was also found that data sheets, Figs. 3–5, which comprise the structural parameters, the final answer and a sketch of the problem, provide a visual aid on the way the external loads are dispersed which simplifies the task of structural detailing. Furthermore, these data sheets serve to standardize the methods of presentation of the calculations and result in neat and concise calculation files.

There are various makes of desk-top computers on the British market, e.g. Hewlett-Packard, Wang, Olivetti, etc., each having a different memory capacity and some special features. However, most of these machines are adaptable to accommodate peripherals such as extended memories of various sizes, printers, plotters, etc.

The price of these calculators ranges from £2,000 to £5,000.

This paper describes the use of the desk-top computers for the solution of standard structural engineering problems. Four examples dealing with multi-span beams, interconnected shear walls, cantilever retaining walls and stressed ribbon foot bridges are presented.

It is shown that these calculators are capable of working out complete design information such as member sizes, bending and shear reinforcement, from the basic knowledge of the structure's geometry and loading conditions.

The calculator used for these programs is a Hewlett-Packard 9100B/9101A System. This machine shown in Fig. 1, comprises a basic 9100B unit, a 9101A extended memory and a 9120A printer. This system has a maximum capacity of 3864 program steps or 280 storage registers and costs a little over £4,000.

Example 1

Multi-span beams

This program works out the bending moments in multi-span beams of up to 12 spans under any combination of distributed and point loads.

The analysis is based on the matrix formulation of the single-cycle moment distribution method[1]. In this procedure the joints are initially considered to be fixed against rotation. The fixed end moments are m_{ij}'s. The structure is then brought to its compatible position by means of balancing moments x_i's applied at the intermediate joints, i.e. joints 1 to $n-1$ as shown in Fig. 2.

It can be shown that these moments may be expressed in terms of the fixed end moments m_{ij}'s and distribution factors d_{ij}'s as follows:

$$\tfrac{1}{2}\begin{bmatrix} 2 & d_{21} & O & O & & O \\ d_{12} & 2 & d_{32} & O & & O \\ O & & & & & \\ & & & & & \\ & & & & d_{(n-2)(n-1)} & 2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \\ \\ x_{(n-1)} \end{Bmatrix} = \begin{Bmatrix} m_{10} + m_{12} \\ m_{21} + m_{23} \\ \\ \\ m_{n-1,\,n-2} + m_{n-1,\,n} \end{Bmatrix}$$

or $\tfrac{1}{2}[d]\{x\} = \{m\}$

$[d]$ is an unsymmetrical banded matrix and hence only the non-zero elements need be stored in the calculator.

Solution of the above equation gives the balancing moments $\{x\}$. After one cycle of distribution of these moments and the subsequent carry-over operation, the final moments M_{ij}'s are obtained by summing up the moments at each side of the joints.

Fig. 2 shows the sequence of operations in this procedure.

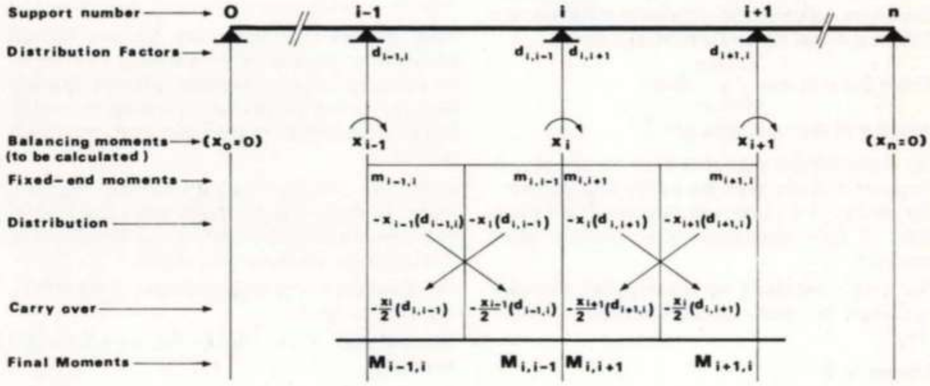


Fig. 2
Sequence of operations in the single-cycle moment distribution method

A complete design of a five-span beam is presented in the standard data sheet shown in Fig. 3. The analysis involved six computer runs for the five load cases to obtain the maximum positive and negative moments and subsequently the computation of the bending and shear reinforcement at the critical sections.

These calculations took 28 minutes.

Example 2

Inter-connected shear walls

This program computes the bending moments, shear forces and deflections along the height of two inter-connected shear walls under uniformly distributed lateral forces.

The calculations are based on the shear connection technique.[2,3]

In this procedure the connecting beams are replaced by a continuous shear medium. The governing equation of this idealized system is:

$$R''-\alpha^2 R+\beta M_0=O$$

where R is the shear in the medium

R'' is the second derivative of R with respect to the distance x, i.e.

$$\frac{d^2R}{dx^2}$$

α^2, β are constants depending on the geometry of the structure, the values of which are given in references 2 and 3

M_0 is the externally applied moment.

The solution of the governing equation is:

$$R=A\sinh\alpha x+B\cosh\alpha x+\frac{\beta}{\alpha^2}\left(M_0+\frac{M_0''}{\alpha^2}\right)$$

where A and B are arbitrary constants which are determined from the boundary conditions.

It can be shown that the moment, and shear forces at any distance x from the top end of the walls are given by:

$$\left.\begin{aligned} M_a&=\frac{wx^2}{2}\left(\frac{I_a}{I_0}-RK_0\right)\\ Q_a&=wx\left(\frac{I_a}{I_0}-R'K_0\right)\end{aligned}\right\}\ \ldots\text{ Wall 'A'}$$

and

$$\left.\begin{aligned} M&=\frac{wx^2}{2}\left(\frac{I_b}{I_0}-RK_1\right)\\ Q&=wx\left(\frac{I_b}{I_0}-R'K_1\right)\end{aligned}\right\}\ \ldots\text{ Wall 'B'}$$

where I_a and I_b are *MOI* of walls A and B respectively.

$$I_0=I_a+I_b$$

$$R'=dR/dx$$

$$K_0=c_a+\frac{1}{I_0}(c_bI_a-c_aI_b)$$

$$K_1=c_a+c_b-K_0$$

c_a, c_b distances from the centre of the medium to the centre of wall A and wall B respectively.

The shear force in the connecting beams at a distance x can be calculated as follows:

$$\text{Shear force at } x=\int_{x-(d/2)}^{x+(d/2)} R\,dx$$

where d is the storey height.

Fig. 4 shows the data sheet for the shear wall program. It comprises the particulars and output results of a 15-storey shear wall with one band of door openings under uniform wind pressure.

The time consumed on the Hewlett-Packard calculator for these calculations was 17 minutes.

Example 3

Cantilever retaining walls

This program furnishes a detailed design of cantilever retaining walls.

The data required are the height of the stem, the width of the base and the properties of the soil.

The program checks the stability of the structure against overturning and sliding. The bending moment and the required concrete thickness, d_1, are then computed for any chosen section along the stem of the wall. d_1 may either be rounded off or completely altered. Having decided on the value of d_1, the program works out the required steel area and prints out these results.

Also, the program calculates the bending moment, concrete thickness and steel area at the critical sections in the base and computes the resulting pressure on the soil.

Fig. 5 shows a typical data sheet for the retaining wall program.

The analysis of the given problem took eight minutes.

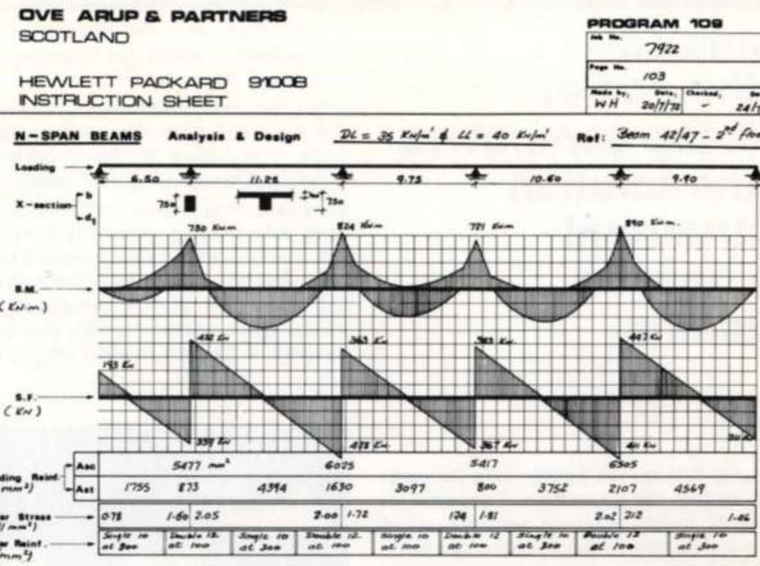


Fig. 3

Example 1 (N-span beams)

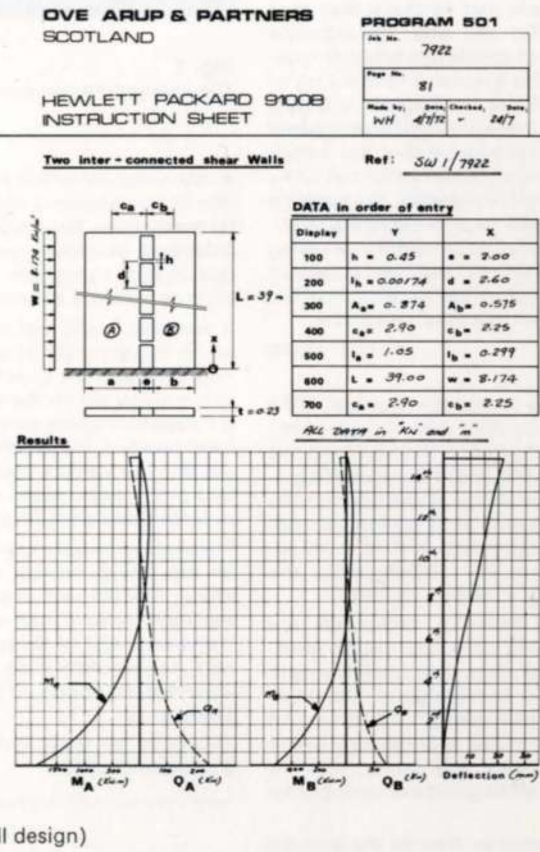


Fig. 4

Example 2 (Shear wall design)

Example 4

Determination of the natural frequency of stressed ribbon bridges

Stressed ribbon bridges[4] consist of slightly sagging, tightly-tensioned cables forming a catenary. These cables support a thin deck which serves to distribute the vertical loads and provide continuity.

The bridges are extremely simple and structurally elegant. Generally they are light and hence their tendecny to oscillate is high, particularly during construction, when the damping devices (e.g. handrails, etc.) are not fitted.

A proposed scheme of a stressed ribbon footbridge in the west of Scotland necessitated a detailed study of the dynamic behaviour of these structures. A program was therefore written to work out the natural frequency of the free, undamped vibration of stressed ribbon bridges.

The idealized structure (shown in Fig. 6) consists of a tensioned, massless cable; T is the force in the wires, and the vertical loads are represented by lumped masses.

It is assumed that the lateral displacements are small and hence the tension force does not change appreciably.

The flexibility equation of this system may be given by

$$\begin{aligned} r_i&=f_{11}R_1+f_{12}R_2+\ldots f_{1n}R_n\\ r_2&=f_{21}R_1+f_{22}R_2+\ldots f_{2n}R_n \qquad \text{(a)}\\ r_n&=f_{n1}R_1+f_{n2}R_2+\ldots f_{nn}R_n\end{aligned}$$

where r_i is the displacement at 'i'
R_i is the force at i
f_{ij} is the flexibility coefficient
n is the total number of masses.

This above equation can be rewritten as

$$\{r\}=[f]\{R\} \quad \text{(b)}$$

but

$$\{R\}=-[m]\{\ddot{r}\} \quad \text{(c)}$$

where $[m]$ is a diagonal matrix of the masses
$\{\ddot{r}\}$ is a vector of the acceleration.

Introducing (c) into (b) we get

$$\{\ddot{r}\}=-[f][m]\{\ddot{r}\} \quad \text{(d)}$$

Assuming a solution for (d) in the form

$$\{r\}=[X]\cos\{\omega t+\epsilon\} \quad \text{(e)}$$

where t is the time
ϵ = arbitrary constants
$[X]$ is a diagonal matrix.

Substituting (e) into (d) we get

$$X=\omega^2[f][m]\ X \quad (f)$$

This above equation is a characteristic value problem, the Eigen values of which represent the angular frequencies and the Eigen vectors are the corresponding amplitudes of the motion.

The Hewlett-Packard program works out and stores the flexibility matrix and employs Stodola and Vianello's iterative method[5] to determine the lowest Eigen value, ω^2. The natural frequency of oscillation, fr, is then computed as follows:

$$fr=\frac{\omega}{2\pi}$$

To verify this computer program, the natural frequency of vibration of the Bircherweid footbridge was computed by using the Hewlett-Packard program and compared with results obtained experimentally on the same bridge and reported in[6].

These values are:

fr theoretically $=1.51$ Hz
fr experimentally $=1.59$ Hz

The data input were merely the span, the tension in the cable and the weight of the concrete deck, as given in Fig. 6.

This analysis took 13 minutes

Conclusion

This paper outlines the advantages of desk-top computers and discusses their utility in design offices.

It is recognized that the usefulness of these machines lies mainly in their high speed, accuracy and their ability to store and manipulate data from which the results can be extracted.

It is these capacities that can effectively be employed to minimize the time consumed in solving manually the standard everyday design problems.

An apparent advantage of using desk-top calculators is that the approximate methods of design, as well as the methods based on empirical formulae, which are often used in design offices, can be replaced by more exact techniques such as matrix analysis and energy methods. Furthermore, a discipline can be established in which calculations done on these machines become an integrated part of the activity in the design office, with the obvious advantage of unified design methods and standard format for the presentation of these calculations.

The four examples given above are selected from a library of nearly 120 programs prepared by Arups, and cover a wide range of structural engineering problems (e.g. beams, columns, frames, section properties, etc.).

Broadly speaking, these examples illustrate that desk-top computers can be used as production tools of complete design information from the basic data of the structural geometry and loading conditions.

However, each of these examples emphasizes a specific point.

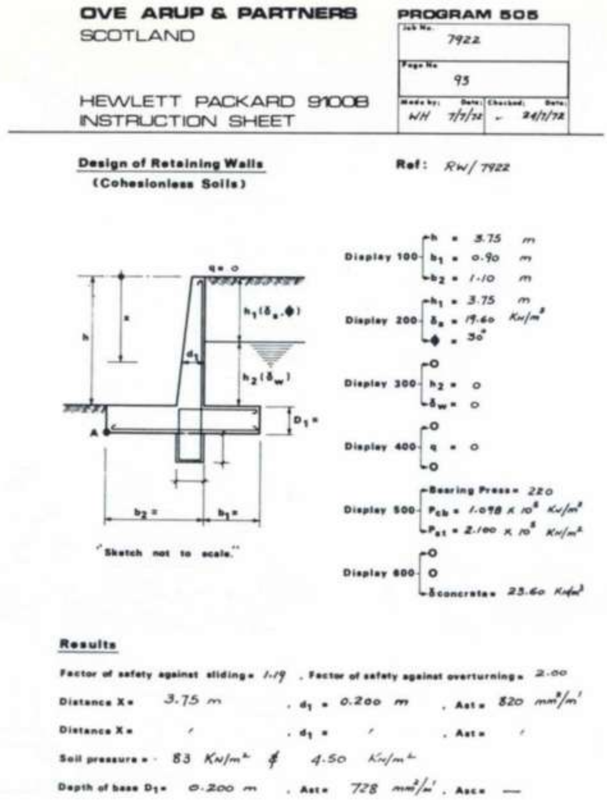


Fig. 5
Example 3 (design of retaining walls)

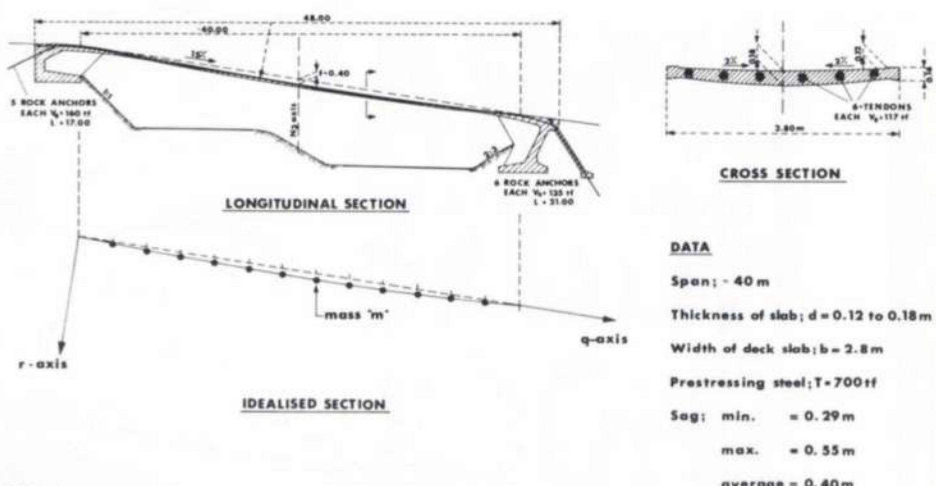


Fig. 6
Example 4 (stressed ribbon foot bridges)

Examples 1 and 2 show that the complexity of refined analytical techniques, which makes their use for hand calculation prohibitive, can be solved by automatic computation.

Example 3, which deals with the design of retaining walls, illustrates that desk-top computers serve to produce economically the optimum solution of the problem. This program assists the designer in sizing retaining walls, and then works out the earth pressure, the factors of safety, the bending moments and steel reinforcement at the critical sections.

Example 4 uses the flexibility method for formulating the load-deflection relationship of stressed ribbon footbridges and works out the natural frequency of oscillation by iterative methods. This example indicates that some supposedly complex problems, which are often referred to digital computers, can readily be dealt with on a medium-size desk-top calculator.

Finally, these machines are accessible to the users. This may well be one of the major advantages of desk-top calculators for, unlike digital computers, programming of these machines is simple and can be done by the user. Also, turnover of the results is usually fast, which allows various alternatives to be examined.

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Born out of conflict
Utzon's peeled orange
Opera theatre
Letter from Sydney
L'Opéra de Sydney trop perméable aux décibels du port!
OPERA HOUSE SHOCK
Govt has no new Opera parking plan
Inside the Opera House
THE MAKING OF The Opera House
The many marvels of
OPERA HOUSE
$220 MIL OPERA HOUSE
Not all plain sailing
What, no chimney stack?
UTZON BACK!
Utzon may be at opening
OPERA HOUSE
MAKING THE IMPOSSIBLE DREAM