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Front and Back Cover: Tracing of principal shadows from a photograph of the mullions at Northwick Park Hospital. (Photo: David Howells)

## Northwick Park Hospital

## Nigel Thompson

## Introduction

Northwick Park Hospital consists of a 600bed district general hospital provided by the North West Metropolitan Regional Hospital Board and a 200-bed clinical research centre provided by the Medical Research Council. planned together as an integrated whole with supporting residential accommodation.
The association of a district hospital and a clinical research centre in one complex is unique in this country and holds promise of many reciprocal benefits. Research workers will be in direct contact with medical problems as they manifest themselves in the community and the General Hospital staff will have access to the advanced diagnostic and other facilities afforded by the Research Centre.
The passage of this project through the various ministerial committees was smoother and faster than most hospitals, as it was planned that this combined centre should attract staff from throughout the country and even, it was hoped, might help stem the brain drain!
The approximate total cost of the project is $£ 15$ million and the site covers an area of 46 acres in north-west London. The architects are Llewelyn-Davies, Weeks, Forestier-Walker \& Bor who were responsible for our appointment as consulting structural engineers in the summer of 1962.
The hospital is divided into three phases of approximately equal value. The work started on site in 1966 and now, after three years, Phase I is nearly ready to be handed over and Phase II is structurally complete. The main contractors. Trollope \& Colls Ltd., had a bad start but in recent months they have built at a rate of just over $£ 3$ million per annum, the 2 fastest rate of hospital building in this country.

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Maternity Unit
Psychological Medicine (Psychiatry) Rehabilitation Unit (Physical Medicine) Library, Lecture Hall
Medical Illustration
Clinical Research Institute
Animal House
Radiochemistry
Isolation Unit (Communicable Diseases)
10. Operating Theatres
11. Recovery Unit
12. North Wing (Clinical Research Centre Wards)
13. East Wing (Medical/Surgical Wards)
14. South Wing (Medical/Surgical Wards)
15. Chapel
16. Clinical Lecture Theatre
17. Staff Dining Rooms
18. Central Kitchens
19. Pharmacy
20. X-Ray: Central Supply Dept. Underneath
21. Pathology
22. Shopping Square
23. Administration: Main Entrance
24. Outpatient Department
25. Accident and Emergencies : Mortuary Underneath
26. Supplies Delivery
27. Boiler House
28. Oil Storage
29. Maintenance Department
30. School of Nursing
31. Staff Residences
32. Common Rooms
33. Staff Houses
34. Flyover

Roads

$\square$ Space for future building
$<$ Indicates direction of future growth.

Fig. 1
Architect's model showing main hospital complex excluding residential area.

Fig. 2
Architect's site plan.


## Structure in principle

The design and planning principles that John Weeks laid down, which directly affected structure, were that the hospital should be designed to accommodate growth and change and that the lines of communication should be clear and simple. At Northwick Park, where there were no limitations or constraints imposed by the site, the architectural solution to these principles was to plan the various departments as a series of carefully linked buildings, at the heart of which there would be the multi-storey blocks of wards and research laboratories. The buildings were separated so that each block or department could be extended or altered individually to meet the changing demands of medical practice.
At the same time as hastening the design process, this separation of blocks would make it possible for part of the project to be opened at an early stage while construction continued on the remainder. Individual buildings were to be connected by a simple two- or three-storey internal street/corridor system: the upper level for use by patients, staff and visitors and the lower levels for service vehicles, trolleys and piped services.
In order that the patient level should be maintained horizontally throughout the major part of the complex, buildings were rotated to suit the natural contours of the site. Also, in order to speed the planning, in addition to easing the provision for growth and change. only like departments were stacked vertically above one another. This reduced, to a large extent, the complicated tangle of services generally associated with hospitals.
At the planning level, therefore, the principle was separation of departments, while at the design level, the principle was separation of trades.
Invariably in hospitals, the main obstacle to change is the inability of the structure to provide sufficient freedom for the movement of services. Our brief, therefore, was to design a structure that was not only cheap and fast to construct, but one that would get out of the way, to give the planner and the services flexibility now and in the future.
The programme, which was intentionally tight ( $£ 6.5$ million of functioning hospital in six years), was open-ended, so that the client could continue writing his brief while buildings were being constructed.

Figs. 3 a-d
Four stages in the planned growth of the hospital.

## 3a

Phase 1
3b
Phase II
3c
Phase III

## 3d

Possible fourth phase
Phase 1 is known as the 'Minimum Viable Unit' and has been designed in such a way that it can open and function as a complete hospital. The main lines of communication are clearly established during this phase and are shown hatched on the plan. The buildings or departments are effectively hooked on to these main routes and grow with the sequential phases.

Arrows at the ends of buildings indicate directions of planned growth.


3b


3d

## Structure in detail

## Substructure

The site consists of London Clay to the surface varying from 44 ft . to 78 ft . ( 13.4 m to 23.8 m ) in thickness, overlying the Woolwich and Reading Beds. The site investigation revealed a greater concentration of sulphates in solution in the ground water than had been recorded naturally elsewhere in the United Kingdom.
In order to limit differential settlements it was decided to found the majority of the buildings on piles, except where deep basements were required when rafts became the more economical solution. Because of the high sulphate content, very dense cast in situ concrete piles of the Franki-type were adopted.

## Superstructure

We decided that, in order to give freedom for the horizontal movement of services at any time in the life of the building, there should be no upstand or downstand beams internally within the blocks. Also, on account of the flexibility business, and because the position of all minor services are invariably unknown until the design process is complete and work has started on site, it was essential to provide for a high degree of vertical perforation of the horizontal structure. We, therefore, favoured for the construction of floors, a plate-type solution with pre-planned areas of permissible penetration.
The entire hospital was planned to a module of $11 \mathrm{ft} .4 \mathrm{in} .(3.45 \mathrm{~m})$. This dimension was considered, at that time, to be an ideal dimen-
sion for the width of a laboratory as well as a single ward.
For the low rise blocks, namely, the Accidents, Out Patients, Diagnostics, Operating Theatres, and Maternity, a $12 \mathrm{in} .(300 \mathrm{~mm}$ ) deep coffered floor slab with 6 in . ( 150 mm ) wide ribs on a module of 2 ft . 10 in . ( 860 mm ) and supported internally by columns spaced at 22 ft .8 in . ( 6.9 m ) centres in both directions, fulfilled all the design requirements and came within the cost allowance. By coffering the plate floors, the self weight was kept to a minimum with the consequent saving in vertical structure and foundations, and the desired grid framework for possible penetration was provided; holes being as easily cut after the slab was cast as formed in the first instance.


Fig. 4a
Typical low-rise block (Accident Department) : internal supporting structure of columns at 22 ft .8 in . ( 6.9 m ) centres in both directions.


Fig. 4b
Ward \& Link Block (multi-storey):
internal supporting structure of short staggered walls $22 \mathrm{ft} .8 \mathrm{in} .(6.9 \mathrm{~m})$ centres to coincide with partitions.


Fig. 4c
Laboratory Block (multi-storey) : internal supporting structure of loadbearing ducts.

In the case of the multi-storey ward blocks. where the floor layouts were already agreed with the client before we were appointed, the only internal vertical structure that worked consisted of short staggered walls. The requirements with regard to a beamless floor still prevailed and, as we needed to span in all directions, the coffered slab again became an economical and satisfactory solution.
For the laboratories which are tall blocks, generally rectangular on plan, with 22 ft .8 in . deep laboratories placed on either side of a double width corridor, the vertical structure was simply provided by supports at the perimeter of the buildings and, internally, by short concrete walls enclosing two sides of service ducts spaced regularly along the corridor. The floors were thus essentially oneway spanning. However, since demountable partitions were planned to a primary module of 2 ft .10 in . and the floors were required to carry very heavy point loads, the coffered floor again became a feasible and economical solution.
Therefore, with the exception of the Boiler House and Incinerator Block, which were structural steelwork with a multi-flue concrete chimney, and the residential area. which comprised seven- and three-storey blocks of loadbearing brickwork with a precast proprietary flooring system, the entire floor area of the hospital was to be structured with a 12 in . ( 300 mm ) deep coffered slab-in fact just over 20 acres ( 9 hectares) of coffers.
All buildings are connected by an underground tunnel system for trolleys and/or services, which takes the form of a single or double decker concrete box culvert. The pedestrian streets which run across flat roofs or follow the line of the tunnels are enclosed by a simple tubular steel portal, clad with patent glazing.
For the elevational treatment of all patient areas John Weeks decided that he would like to use concrete mullions in order to visually link the different blocks together and, for all extendable ends, decided that the structural cross-section should be simply left exposed. so that when extensions were added, an expansion joint would be formed and then the building would march on. Plant rooms, where they occur on roofs, are also clad with patent glazing.
In a combined effort, after trying many different schemes, we finally produced a


Fig. 5
Architect's elevations of multi-storey and
low rise blocks. The mullion spacing
emphasizes the different loading conditions.

adj. of cybernetics
-a science of
control and
communication in complex electronic machines like computers and the human nervous system.
the faculty of making happy chance discoveries.

Cybernetic Serendipity

In an exhibition with this title at the Institute of Contemporary Arts at Nash House, an exhibit entitled 'Indeterminate dimensions in architecture' showed the Northwick Park Mullion System. The system was eligible for display because the geometry and spacing of the mullions, and hence the ultimate appearance of the system, were determined wholly as a result of engineering analysis and consideration of practical building problems. A computer could have been used but, no suitable program being available, it was decided to proceed manually as it might well take longer to write a program than it would to individually design all the elevations.
In so far as it was logical to say that we should use a 12 in . by 6 in . precast concrete
mullion to act as the loadbearing member for the perimeter of all buildings, regardless of height or loading, the system is logical. Instead of increasing the size of the column as it gets nearer the ground, or increasing the quantity of reinforcement as in traditional structures, we decided to increase the number of mullions. We have 4,000 identical mullions which we have positioned so that, as far as possible, the loads equate. Thus, to a certain extent, the loading of the interior is expressed on the elevations. Mullions cluster below areas carrying heavy plant or large interior spans while they fan out for the more lightly loaded conditions. We have 2.500 beams with approximately 20 different types: variations being caused by length, end condition.
internal corner, external corner, etc. Basically, we use the same beam in all conditions. A pocket is always provided on the top and soffit of the beam and these pockets are either filled with a mullion or left exposed.

The size of the beam being determined by height of skirting and depth of false ceiling. was sufficient, with a nominal cage, to spread the loads horizontally. The mullions are spaced on a primary module of 2 ft .10 in . ( 860 mm ) coinciding with the internal coffered rib grid and the glazing or weatherproofing wall is placed inside the mullions with a 3 in . ( 75 mm ) cavity so as to divorce the windows from the spacing of the mullions.



Fig. 6
Isometric showing
details of the precast
mullion system.


Fig. 8
Mullion occurring above only.

Figs. 8 to 10
Sections through the beams.

In order that the number of mullions should be kept to a minimum at the top floors and the maximum horizontal spread of load should be achieved, it was essential to provide a degree of continuity between beams. For simplicity in erection and practical considerations of handling, it was desirable to have a mullion beneath all beam joints.

Now to provide continuity between two precast beams over the top of a mullion, as well as producing a satisfactory watertight seal. we found it necessary to stagger the beam joints so that a mullion always occurred below a beam joint but never above. This device greatly simplified detailing.
The resulting system is an amusing and interesting architectural idea which has been produced quite cheaply.


Fig. 10
Mullion occurring below only.


Fig. 11a


Fig. 11b


Fig. 11c

Figs. 11a to $\mathbf{c}$
These show the change in elevation as the camera moves through $50^{\circ}$.

## Construction

Substructure
Before the commencement of contract piling. six preliminary test piles were installed and tested. They performed very satisfactorily and confirmed the design previously agreed between Frankipile Ltd. and ourselves.
Installation of working piles began on 1 March 1966. After three weeks-some 600 piles later-a working pile failed at half the load of the equivalent preliminary pile. Subsequently. seven other piles failed in a similar manner. In the meantime piling continued at an increased length. The longer piles, when tested, again failed. The piles were downgraded in loadbearing capacity and even longer piles and additional piles were driven. A detailed investigation was carried out which included considerable testing, the exhumation of two piles and the re-testing of the preliminary piles.
In our view the results from the investigation indicated that the failure of the working piles was due to the way in which they were made. The Building Research Station endorsed this view. However, no indisputable technical conclusion could be reached, and certainly the strange behaviour of the piles could not have been foreseen before or during their installation. Needless to say, a contractual storm developed. Despite the difficulties, piling finished five weeks ahead of schedule. Credit for this must go to Frankipile Ltd., for the speed with which they carried out the remedial piling and finally completed their work on site.

Fig. 12
View of four short piles being exhumed. They were installed by Frankipile Ltd. after the failures in order to test the effects of different methods of installation.

Fig. 13 A covered pedestrian street entering the ward block



4a

Fig. 14a
Area of coffered slab prior to the installation of services.

Fig. 14b
Interior shot of the hospital after the installation of services.

## Superstructure

To construct the 20 acres ( 9 hectares) of coffered slab, the main contractor decided to completely deck-out all floors with a ply shutter and use purpose-made removable fibre-glass moulds to form the coffers. He found that, contrary to first thoughts, it was cheaper to deckout rather than use skeletal formwork, despite the theoretical saving in time claimed by advocates of the skeletal method.
The pans were removed by either (a) a mechanical device which consisted of two nuts cast into the mould through which bolts were inserted to jack the pan free from the concrete. (b) by compressed air, or (c) by brute force and wedges. The most successful was the compressed air method, providing the pans were tacked to the decking to prevent grout loss. Where pans were not held down. the brute force method had to be adopted which tended to reduce the life of the pans. The estimated re-use factor of the moulds has been between 5 and 12 times. The contractor later carried out an experiment using cardboard formers for an area of approximately 3.000 sq. ft. ( $280 \mathrm{~m}^{2}$ ). This proved unsatisfactory, as far as he was concerned, with regard to cost, speed and durability and, as far as the architect was concerned, on account of appearance.

The precast mullion system which supports the edge of the floors for most blocks around the hospital has been erected simply and with reasonable speed.
And now, at the end of Phase 1, as the handover date approaches, turf has suddenly appeared outside some of the buildings. This has had the usual heartening effect of making the concrete look as if it has been dry cleaned-an exciting experience after $61 / 2$ years on this project.

Fig. 15
Contrasting elevations caused by the junction between the Clinical Research Wards (Block 12 on Fig. 2) on the left hand side of the photograph, and the Clinical Research Laboratories (Block 6 on Fig. 2).
The finish on the precast concrete mullions. which were cast in steel moulds, was achieved by acid etching. The granite aggregate is thus lightly exposed on all faces. A similar texture has been achieved on the front face of the beams, which were cast in plastic faced plywood moulds, by exposing the fines through water washing. The fine aggregate was a Lee Moor Sand.
All precasting which was of a consistently high standard, was carried out by Trollope and Colls at their Camberley works.


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Fig. 16
South elevation of the Accident Department (Block 25 on Fig. 2), showing the ambulance ramp, a vent shaft, and an external escape staircase.
The Accident Department is raised off the ground to link up with the main hospital level. the space below providing necessary car parking accommodation.



Fig. 17
East elevation of the Outpatients and Accident Departments (Blocks 24 \& 25 on Fig. 2). This elevation is effectively the heel of these departments which expand in a westerly direction.
The photograph also shows a street cantilevered from the Outpatients Department with a link across to the Pathology Department (Block 21 on Fig. 2). The patent glazing box on the roof is a plant room.


Fig. 18
General view from the east showing the multi-storey ward blocks.
Fig. 19
View of the Institute Building with the Animal House under construction on the right hand side.

Fig. 20
General view from the west of Phase I showing the extendable ends.

Fig. 21
View of the link block between the operating theatres (low rise block on the right hand side) and the main ward complex.

CREDITS
Fig. 1 Photo: Sydney W. Newbery.
Figs. 2, 4 \& 5 Drawings supplied by Llewelyn Davies, Weeks, Forestier-Walker \& Bor.

Fig. 3 Drawn by: Jenny Wheatley.
Figs. 6 to 10 Drawn by: Andrew Actman.
Figs. 11, 13-15, \& 18-21 Photos: David Howells.

Fig. 12 Photo: J.S. Kent, Resident Site Architect.

Figs. 16-17 Photos: F. J. Hare Photographers.

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## The analysis of the Al viaduct, Gateshead

## Bill Smyth

and
Srinivasan

## Introduction

The construction of the A1 viaduct started on site in Gateshead in March 1969, our first urban motorway in this country, one of the most complex structures we have designed and the culmination of years of effort by a devoted team
We could easily write a book on the design of the viaduct, on the tests we made, the foundation problems, the shuttering, the lighting, the amenity problem and much more,
but this article deals with the analysis. so that to set the scene we will just start with a few vital statistics.
Fig. 2 gives some idea of the complexity of the viaduct and the main dimensions are :

Overall length
of dual carriageway
including approach ramps: $3130 \mathrm{ft} .(954 \mathrm{~m}$ )
Length of suspended
deck, excluding ramps: $2112 \mathrm{ft} .(645 \mathrm{~m})$
Length of suspended ramps: $562 \mathrm{ft} .(171 \mathrm{~m})$
Typical span:
$88 \mathrm{ft} .(27 \mathrm{~m})$
Maximum span (two ramps) : $105 \mathrm{ft} .(32 \mathrm{~m})$
Normal width
of main viaduct: $\quad 64 \mathrm{ft} .(19.5 \mathrm{~m})$
Maximum width
of main viaduct: $\quad 124 \mathrm{ft} .(38 \mathrm{~m})$


1



Fig. 1
1:500 model of viaduct. (Photo: Henk Snoek)
Fig. 2
Plan and elevation. Elevated structure shown shaded in plan. (Drawn by: Barbara Crebert and S. Sladeng)


ELEVATION


The library has copies of our report to the client on the project design, which shows that we investigated many possible schemes. and that the very considerable constraints of the site, the economic, structural and aesthetic factors, led us to two possible solutions -one in steel and one in prestressed concrete, of which the prestressed concrete one was the cheaper by an estimated $£ 140.000$ or about $16 \%$ of the structural cost. The prestressed concrete solution was obviously sensible in structural form, but difficult to analyze. We had to be able to satisfy ourselves that our structure would be safe, but we did not feel that analytical difficulties should stop us from designing the right structure for the situation. We believe that the cost figures tell their own story.
In our preliminary estimate in the report of June 1966 (i.e. before devaluation) the total estimated cost of the structure, including foundations, barriers, surfacing and drainage was $£ 1.042,000$. Our 3 X estimate of November 1967, for the grant application to the Ministry of Transport, gave a comparable figure of $£ 1,123,100$. The tender figure in October 1968 was $£ 1.004 .542$ or $£ 5.50$ per sq. ft . ( $£ 59.2$ per $\mathrm{m}^{2}$ ). with the structural deck costing $£ 4.01$ per sq. ft . ( $£ 43.1$ per $\mathrm{m}^{2}$ ). The total contract price (including a great deal of road work at ground level) was $£ 2.390 .000$. Tendering was very close, with the two highest of the six tenders coming out just above our final estimate.

## Preliminary analysis

During the preliminary design we used very crude analyses to satisfy ourselves that the concrete sections were adequate and to obtain reasonable estimates of prestressing and reinforcement. The slip roads were ana-
lyzed as folded plates with hinged joints which, for a triangulated cross-section. produces an accurate enough solution for the purpose, provided that allowance is made for the secondary bending moments, in the same way as in a truss with rigid joints. For the main structure we assumed that the crossheads were infinitely stiff and started by analyzing a rectangular box of about the same proportions, using the Cement and Concrete Association's tables based on the MassonetGuyon distribution method. The actual trapezoidal cross-section of the main structure was dealt with by putting the two analyses together with engineering judgement, better known in other contexts as woman's intuition. We expected that this crude analysis would give us too much prestressing in the crossheads, slightly too little in the longitudinal direction and that we would be able to make a reasonable estimate of the total amount of prestressing required.
The most complicated structures occur where the slip roads join the main structure and here, not only was the elastic analysis difficult, but the structure could not be shown to be stable against overturning by treating each individual bay as a separate entity, and it was necessary to take several structural bays acting together. Again a very crude analytical model was used, based on an equivalent beam system.
Final analyses-general considerations Very crude methods have often been used to analyze box sections, such as applying an arbitrary and high proportion of the total live load on to each individual web and then treating the structure as a series of I beams. This is very economical in engineering effort and saves time, but it makes no use of the excellent load distributing properties of the
boxes and tends to make them appear uneconomical in relation to other forms of construction. It was obvious that we would have to use more sophisticated analytical methods, because the considerable ratio of breadth to span made the structure comparable to an intermediate length shell rather than a long shell or beam, and the flexibility of the crosshead was of the same order of magnitude as the longitudinal flexibility.
Any method of analysis which is too crude in the particular circumstances produces a structure which is either unsafe or too expensive. To satisfy ourselves that we were really designing an economic structure, making full use of the load distributing properties of the section, we had to attack the problem more fundamentally and to try to understand how box sections really work.
Another factor affecting the analysis of this kind of structure is the large number of loading cases which has to be examined to ensure that it satisfies the Ministry of Transport's standards: a too complex analysis could easily take several years, and we could spend vast sums on computer time and end up with neurotic engineers, without even necessarily having arrived at a better understanding of the way in which the structure works.
Two kinds of analysis were required: one, the simplest possible satisfactory method of. analyzing the structure in order to deal with a number of loading combinations and the difficult problems involved at the slip road junctions: the other a more sophisticated analysis to check and, if necessary, calibrate the simpler one. We also hoped to get a deeper understanding of the action of box structures.

c-c

Fig. 3
Plan \& sections of southern ramp
intersection. Plan drawn with top deck
removed to show ribs. (Drawn by: Barbara Crebert)


Shear stress diagrams.


Longitudinal stress diagrams.
Note: No longitudinal stress in System 2. System 3 shows warping stresses.


Deflections due to each force system. System 1 produces pure vertical deflections. System 2 produces pure twist. System 3 produces 'diamonding' of cross section as shown, as well as longitudinal warping.


Effect on deflections if box has rigid joints at corners instead of hinges. Distortion of cross section in its plane ('diamonding') considerably reduced as shown. Distortion out of plane (warping) correspondingly reduced.

Fig. 4
Single cell rectangular box with diaphragms at supports only.

At this stage, we made a list of all the probable and improbable ways we could think of for tackling the analysis. In some ways the problem is analogous to that of a beam on an elastic foundation except that it is rather more complicated and, as with shells, we are in an intermediate situation where one can neither start from a completely rigid beam and then correct the solution, nor from a completely flexible one. The problem has to be tackled as an entity and this rules out a number of approaches which might otherwise have been possible.

## General studies

In order to try to gain some more fundamental understanding, or feel, of the way in which closed box structures work, we looked at the actions of various simple one cell, two cell or three cell rectangular and triangular boxes. We started with rectangular boxes which were hinged at the corners so that the corner joints could transmit longitudinal shears but could not take any transverse bending moments. The result showed that even a very flexible box can have excellent transverse distribution properties and can deal very well with torsional loads, but at the expense of considerable deformations. (See Fig. 4)
When we looked at the effect of making the corner joints stiff we found that, within the range of possible prestressed concrete structures, the transverse bending stiffness of the rectangular boxes is at least as significant as the other actions and in most cases is probably more significant. This implied that an equivalent grillage analysis might give reasonable results.

## Simple analysis

The simplest adequate method of analyzing the structure seemed likely to be an equivalent slab. or an equivalent grillage. The equivalent slab approach was not satisfactory because of the trapezoidal cross section and the flexible crossheads. We therefore decided to attack the simple problem with an equivalent grillage. This was complicated by the fact that to begin with we didn't know what properties to give to the grillage. Even if the warping of the cross section did not have a significant effect on the transverse moments, the section acts transversely as a kind of Vierendeel girder and, in order to simulate the structural action, the grillage analysis had to take into account, not only the bending deformation of the transverse beams in the grillage, but also the shear deformation, or rather pseudo shear deformation.
In a solid beam the shear deformation is normally not significant but a single beam representing a Vierendeel girder has to have shear and bending deformations represented. The equivalent beam appears to have a shear flexibility which is out of all proportion to its bending flexibility. This pseudo shear flexibility really represents the bending of the top and bottom flanges of the Vierendeel girder. To deal with this, our computer program for grillage analysis was re-written to take shear deformation into account: and that. dear reader, is how our grillage program came to have its shear. To represent the cross section by a member with pseudo shear properties also requires pseudo bending properties and some members have negative moments of inertia (believe it or not).

## Complex analysis

In order to check and possibly calibrate the equivalent grillage there were two groups of approaches-one analytical and the other using physical rather than conceptual models. Physical models had obvious attractions but the amount of information which could be obtained from them was limited, and the time scale required to produce and test the models made them useless for design purposes. In the end, two physical models were used, one as a final check on the elastic analyses of the more difficult parts of the structure at the ramp intersections, and the other as part of a

#  

Bending deformation
(extension, contraction and bending of flanges).


Pseudo-shear deformation
(bending of flanges accompanied by some bending of webs).

Fig. 5
Transverse sections through one cell of multicell box showing deformations represented by transverse bending and by transverse shear of grillage. (Drawn by: Barbara Crebert)
research project in which we were participating.
A number of analytical methods was investigated and rejected for one reason or another. So that the general reader will not be deterred at this point they are listed in a later section with comments to show why we didn't use them. This was usually because they are highly inaccurate, or involve too much computational effort, or involve writing new computer programs which could not have been done in time. It should be remembered that this was three years ago.
The method which we adopted could make use of existing computer programs and the deficiencies of the conceptual model could readily be seen because they arose from physical assumptions, rather than mathematical approximations, and so could be taken more easily into account when translating the results back into designs for the real structure.
Our conceptual model is based on two main assumptions. The longitudinal action of each individual plate between node or junction lines is represented by simple beam theory. in other words straight lines between nodes remain straight. This seems to be a satisfactory assumption for plates with the proportions of those in our structure. A more radical assumption is that the plates are not connected continuously along their edges but at a number of discrete points, at which all forces and moments are transmitted. As the number of these points increases, the model approaches nearer to the actual physical conditions, so that we had to establish how many points were required for a satisfactory solution. To do this we took some very simple cases such as I beams or single cell boxes, for which the results were already known, and analyzed them with various numbers of connections along the span.
Once this discrete point connected model is assumed, it can be accurately represented by a space frame for which there are readily available computer programs, which was at the time an enormous advantage. The space frame model is analogous to the plane frame model which Duncan Michael used for shear walls in buildings. Each plate can be represented in a longitudinal direction by its properties as a beam, concentrated on its centre line. A series of rigid outriggers at right angles to the central line connect it to the edge of the beam at the node points where the plates are supposed to be connected.

These outriggers represent the property plane sections remain plane, so for the given assumptions the ends of the outriggers will have the correct deformation in the longitudinal direction at the node points on the outer edges on the corresponding beam. This produces a bending moment diagram for the plate which has steps in it because the shears between the edges of adjacent plates are applied at the node points instead of continuously along the outer edge, but it is not difficult to draw a reasonably accurate bending moment diagram through the steps, provided that they are not too large in relation to the maximum value of the bending moment. In the transverse direction the outrigger has to be given the bending stiffness of a width of plate corresponding to the distance between outriggers. Again this produces transverse moments which, if plotted longitudinally, will give a stepped diagram.
We now have a model whose physical implications are quite clear and which can be analyzed using already available computer programs.
With enough node points to give a reasonable representation of the structure the space frame becomes rather large, so it was only used to check a limited number of loading cases on the straight spans for comparison with the corresponding grillage analysis.
The comparison indicates that the grillage has to be modified, to represent transverse actions in particular.

## Problems in communicating with the computer

In both the grillage analysis and the space frame analysis there was a great deal of work to be done in putting the data into the computer in the first place and in transforming the output into terms of stresses which could be used for design purposes. This problem is particularly acute with prestressed concrete structures in which the prestressing is itself a loading case. This rubs in something that we have realized for a long time, that the really critical problem with our electronic allies is to find ways of generating the input so that the engineer fills in a small amount of data instead of a large amount, and of representing the output in such a form that the important information is readily available and doesn't have to be picked out with great labour from a vast jumble of numbers. We have already tackled this problem and for slab bridges we are well down the road to a satisfactory solution.


NORTHERN RAMP INTERSECTION SPANS 4-9 2

Fig. 6
Computer plot to check geometrical input
for grillage analysis. (Output by Elliott 4120)

Fig. 7
Development of space frame conceptual model. (Drawn by: Grete Lund)


Elastic lines have properties of plates they represent.
Outriggers are infinitely stiff in longitudinal direction to represent plane sections remaining plane.
Outriggers have properties of plates they represent in the transverse direction.


Cross section of space frame representing single cell box

Cross section of space frame representing main viaduct deck.


Fig. 8
Perspex model at Wexham Springs. (Photo by courtesy of the Cement \& Concrete Association)


Fig. 9
Comparison of longitudinal stresses from grillage analysis with those from perspex model. (Drawn by: Barbara Crebert)
No transverse stress results were obtained from model.


Grillage analysis results shown by hatching. Perspex model results shown by solid line.


Fig. 10
Section of plaster model after failure. (Photo by courtesy of Professor Brock, Loughborough University Civil Engineering Dept.)

## Physical models

## 1. Elastic Model

We have said that the conceptual model of the equivalent grillage was checked for typical straight spans by comparing results with those from the space frame model. The intersections of the slip roads with the main viaduct are too complex to be checked by this means and a physical model was made of one of them. This model took too long to prepare and to test for it to be used for checking the results from the grillage analysis before most of the working drawings were completed: but we had sufficient confidence in the analysis to take our chances on this, as long as we had final confirmation before the structure was built.
The model was made and tested by the Cement and Concrete Association at Wexham Springs. Because of space limitations, we had to make a difficult decision between the alternatives of having a sufficient length of structure represented and having a large enough scale to give us more information. A 1:48 scale model of the structure was made in perspex to represent five spans of the actual structure. 96 electrical resistance strain gauges were installed on the model and the strains were recorded automatically by electronic logging equipment, which unfortunately set an upper limit to the number of gauges.

## 2. Ultimate Load Model

Other tests were made at Loughborough University under Professor Brock to examine the ultimate load capacity of the structure, as part of a research programme to which we were contributing.
Models to a scale of 1:24 were made of one span of the viaduct with a cantilever at each end, to be able to simulate the effect of continuity. The models were made from a plaster with similar characteristics to those of concrete, reinforced with wires and mesh conforming to the reinforcement details of the structure. The prestressing cables were represented by unstressed steel. The self weight and uniform live loads were applied by springs and the abnormal HB load by a special rig on top of the model. Strain gauges were fixed on the top and the bottom along the ribs at the mid and third points of the span and the reactions were measured by load cells at the supports. The load was applied in increments of a quarter of the design load and the models behaved elastically for load cycles up to threequarters of the design load (don't forget that prestress was not simulated as such). The model collapsed at just over twice the design load by the ribs failing in shear, due to effects which would not occur in the real structure. The results from the tests showed fair agreement with the analysis and as the tests were carried out quickly and we had the information at an early stage, this gave us a good deal of reassurance.

## 3. Actual Structure

The best model of all is the structure itself. but it can only be tested under uncontrolled conditions. However, it is intended to instrument the actual structure with acoustic strain gauges cast into the structure.

## Other analyses

So far, we have discussed the various kinds of models which were used for analyzing the behaviour of the structure under applied loadings. There were, of course, many other analyses which had to be carried out. We changed our project design as a result of analyzing the behaviour of the structure under temperature movements because we found that the cumulative stresses in the structure, due to the considerable distance (and consequent load) between expansion joints and thrust abutments required the use of so much extra prestressing that it was considerably more economical to put in extra joints, contrary to our initial instincts.


Fig. 11
Sequence of structural analysis.


TYPICAL CURVED SPAN
WITH FOUR BEAMS
WITH FOUR BEAMS


Fig. 12
Typical computer print out and, above, key drawing giving setting out of deck. (Drawn by: Susan Pickard)

We also had extremely difficult problems in preparing our drawings for setting out the structure on site, and special computer programs were written for this. The setting out of the deck is communicated to the contractor by means of computer print-out read in conjunction with key drawings. We also wrote a computer program which gives the profiles of prestressing cables and the loads imposed by them, which were automatically fed into the grillage analysis.

Appendix : Possible Methods of Analysis Some of the methods which we thought of for analyzing the structure were:

1. Equivalent slabs.
2. Equivalent grillage.
3. Analysis as a simple beam with corrections for transverse flexibility and warping.
4. Analysis as a stressed skin structure using a program developed by Argyris.
5. Three dimensional analysis using finite elements.
6. Analysis using generalized displacements at nodes as outlined by Vlasov.
7. The space frame analysis which has already been described.
8. An extension of this analysis using finite elements in which each element is the full depth of the plate of which it is a part.

Methods 1 and 2 have been discussed and 3 is only suitable for long slender structures which could normally better be dealt with by 6 The membrane solution 4 developed by Argyris is really more suitable for aircraft structures and would probably not give reasonable accuracy with structures whose members have significant bending stiffness. Apart from the modifications which would have been necessary to adapt this method to our problem, a very large computer is required. The program has the attraction of generating the analytical model with a minimum of input data. The finite element method 5 requires either a three dimensional assembly of two dimensional elements as in the Argyris program or preferably a three dimensional assembly of three dimensional elements. At the time when we were carrying out this analysis there were no suitable methods available, and a computer with a very large capacity would have been required.
For our more recent structures we have been working in conjunction with Dr. Chapman at Imperial College. The finite element program developed by Mr. Lim, one of his students, who is subsidized by Freeman Fox, seems to have possibilities but it is only available for the IBM computer at Imperial College and it requires a machine with a very large capacity. The generalized displacement method of Vlasov has a number of very elegant features particularly the concept of the bimoment or self equilibrating set of forces. Vlasov makes the same assumption as we do in our space frame analysis that straight lines between node points remain straight when the structure deforms, but the additional assumption that the members are transversely inextensible would not be satisfactory for a wide structure like ours. The method could be adapted to allow for the transverse extensions at the expense of increasing the number of unknowns. It becomes very involved for a structure with varied cross sections which is continued over several spans and subject to arbitrary loading.
The space frame analysis has already been dealt with, and method 8 is really the next logical step from this method in which. instead of the plates being regarded as connected only at node points along the span, either stresses or deformations are regarded as being continuous between node points. defined by some function such as a straight line. In practical terms, this means that instead of having continuously curved bending moment diagrams, the space frame analysis gives us a stepped diagram but

method 8 would give us a diagram in which the curve is approximated by a number of straight lines without steps.
With the present generation of computers and programs, we would prefer to be able to use the equivalent grillage method or, for structures with different proportions, perhaps an equivalent beam method and we are carrying out general studies in the hope of being able to define the conditions under which the simplified methods can be used.

Fig. 13
Reinforcement and shuttering for main viaduct column. (Photo by: R. Knibbs)

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Client: J. L. Hurrell Esq. Borough Planner \& Engineer, Gateshead.

## 41/2 in. loadbearing brick wall construction at the Edinburgh College of Domestic Science, Clermiston

Peter Carr

Loadbearing brickwork is a structural medium that has, with a few exceptions, been neglected by engineers for many years. Better knowledge of its properties is resulting in it being increasingly used again, but even so, its potential is not fully exploited.
On the continent, particularly in Switzerland which lacks a native steel industry, structural brickwork is widely used for tower blocks up to 20 storeys. In this country the first project approaching this standard was a 12 -storey block of flats at Pershore Road, Birmingham (Ref. 2). Several blocks up to 10 storeys have been constructed, including two designed by this firm: the hostels at Crystal Palace National Recreation Centre (Job no. 850) and the Market Square Redevelopment in Jarrow (Job no. 1769). These all use either 9 in . ( 230 mm ) and $131 / 2 \mathrm{in}$. ( 340 mm ) standard brickwork or single leaf walls in $63 / 4 \mathrm{in}$. $(170 \mathrm{~mm})$ Calculon bricks. The first multistorey buildings using $41 / 2 \mathrm{in}$. ( 115 mm ) loadbearing walls were the residential halls at the University College of North Wales. Bangor (Ref. 3) built in 1964.
When used in the right context, loadbearing brickwork can be very economical. Its major drawbacks are the requirement of a skilled labour force and the difficulty of industrializing the construction process.

Job history
The Edinburgh College of Domestic Science has for many years occupied Atholl Crescent in the centre of Edinburgh. When further expansion there became impossible, the house and grounds at Clermiston, on the western outskirts of the city, were acquired as the site for a complete new college to accommodate 900 students, of whom 450 would be resident. The design was the subject of a competition won by Andrew Renton and Partners (now Renton Howard Wood Associates). The construction of the new college is financed by the Scottish Education Department. The contract was negotiated with Gilbert-Ash (Scotland) Ltd. and has a total value of $£ 1.9 \mathrm{~m}$. of which the structure
accounts for approximately $£ 350,000$. Preliminary design started in January, 1966, the contractor was selected in June of that year. and work started on site in August 1967.

## Design

The winning design consists of a teaching wing, an administration wing and a communal area grouped in a cluster, and three separate residential blocks. The latter are Lshaped on plan with each wing measuring approximately 170 ft . long by 35 ft . wide ( $52 \mathrm{~m} \times 10.7 \mathrm{~m}$ ). Block C contains four bedroom flats grouped around five staircases, mainly four storeys high but with a five-storey section in one wing to allow for the slope of the site. Block B contains study bedrooms arranged in groups about a central corridor,

1


Fig. 1
View of junction of two wings of B block. (Photo: Peter Carr)

Fig. 2
View of B block in foreground and A block in background. (Photo: Peter Carr)
and rises four storeys throughout. The upper three floors of Block A are similar to those in Block B, but on the ground floor there are larger rooms and the structural system changes. The resulting cellular nature of the blocks, coupled with the necessity to keep within the very tight cost limits set by the S.E.D., suggested the use of flat slabs on loadbearing partition walls. The generally small spans involved and the storey height of 8 ft . $6 \mathrm{in} .(2.6 \mathrm{~m})$ allowed the use of 6 in . $(150 \mathrm{~mm})$ slabs and $41 / 2 \mathrm{in}$. ( 115 mm ) brick walls. (See Fig. 3). The teaching wing and administration wing also have a corridor and crosswall layout with flat slabs, but on a larger scale. Although loadbearing brickwork was considered for these, the more flexible planning
requirements made concrete columns with blockwork infill more economical.
The only similar structure we found when we started a detailed investigation of $41 / 2 \mathrm{in}$. brick walls was the college at Bangor mentioned above. This also consists of study bedrooms arranged about a central corridor rising five storeys. As so little data was available at the time, the design of the walls was based on tests carried out on full size sample walls by Professor A. W. Hendry, then at Liverpool University. Strain gauges were fitted to typical walls in the actual building during construction to observe their behaviour and relate it to the experimental results.
Further research has continued under Professor Hendry at the Ceramics Research Unit
at the University of Edinburgh Civil Engineering Department and at the Building Research Station. Experiments have been carried out at Edinburgh on a $1 / 6$ scale model of a single leaf $41 / 2 \mathrm{in}$. wall four storeys high, between concrete slabs to determine shear strengths. At present, a full size 'tower' block, five storeys high using $41 / 2 \mathrm{in}$. brick walls, is being built in a disused quarry and a variety of tests will be made on this.
After reading the available literature, the actual design of the walls was quite straightforward. We first decided that only one brick/ mortar combination would be used at any one level in a block. For the internal walls, it was then a question of selecting the most highly stressed walls at a particular level, and
 Residential B block. Typical floor plan and part-perspective. (Drawn by: Roy Butler)
checking the axial stress against the permissible stresses in C.P. 111 (1964), Table 3. with the appropriate reduction factors applied for the slenderness ratio of 17 , eccentricity of load and, where necessary, for sections of wall with small plan area. It should be noted that tests on full size walls between concrete slabs suggest that the Code allows a load factor of between 5 and 10 (Ref. 4). In particular the effects of eccentric loading appear to be exaggerated (Ref. 5). In these blocks, the only walls significantly affected by eccentric loading are the gables which are $11 \mathrm{in} .(280 \mathrm{~mm})$ cavity walls with the outer leaf of a reconstituted granite block, Fyfestone. However, the stresses resulting from the load applied to the inner leaf only are within permissible limits.
The walls also had to be checked for the effects of exposure 'D' wind loading. The stresses induced in the crosswalls by a wind parallel to them are very small, approximately 5 p.s.i. $\left(0.035 \mathrm{~N} / \mathrm{mm}^{2}\right)$ and the L-shaped plan meant that each block would be stable as a whole. However, we also checked the longitudinal stability of the individual wings of each block. In Blocks A and B there are generally a large number of longitudinal corridor walls which easily take the loads from a wind parallel to them. In Block C there are fewer continuous lengths of longitudinal wall, and the resulting stresses are nearer the permissible limits.
Initial results from the research under Professor Hendry and others suggests that the allowable shear stresses in Section 317 of C.P. 111 (1964) are conservative, particularly when vertical stresses are high. A new edition of C.P. 111, now in the draft stage, is expected to take many of these findings into account
Although satisfied with the final stability of the blocks, we realized that certain sections could be unstable during construction. We therefore specified that they should be built from the corner of the ' $L$ ' outwards along the two wings. This fitted in with the contractor's proposal to build them using a materials hoist at this corner. Individual walls would also be unstable until restrained by the slab above and, even if they did not blow over, there was a danger of breaking the bond between bricks and mortar. We therefore also specified that all walls were to be propped as soon as they were built.
The calculated maximum compressive stress at the lowest level of the four-storey sections was 170 p.s.i. ( $1.17 \mathrm{~N} / \mathrm{mm}^{2}$ ) for which we specified a Grade 4000 brick in a $1: 1 / 4: 3$ cement/lime/sand mortar. At the lowest level of the five-storey section in Block $C$ the maximum stress was 180 p.s.i. ( $1.24 \mathrm{~N} / \mathrm{mm}^{2}$ ) which required a Grade 5000 brick in the same mortar. Grade 4000 bricks are only slightly more expensive generally than common bricks and so we specified the use of these everywhere except at the lowest level of Block C.
To solve a local engineering and architectural problem at the stairwells we required a wall 6 in . ( 150 mm ) thick. The solution adopted was to build walls of two leaves in stretcher bond using bricks laid on edge and tied together with stiff I-shaped galvanized steel ties (Ref. 9).

## Specification

As we did not have a standard specification for loadbearing brickwork, Keith Ranawake had to write one (Ref. 7). Fortunately the British Ceramic Research Association was at that time preparing a model specification (Ref. 6). This was to be published after the final bill of quantities was required, but we were allowed access to the draft. The specification covers all aspects of the selection of materials, quality control of the mortar and standards of bricklaying. It endeavours to 8 apply, where possible, similar standards to


Fig. 4
6 in. loadbearing wall in stretcher bond with bricks on edge. (Photo: Peter Carr)
brickwork as are normally applied to concrete.
The quality of bricks is most easily controlled and is generally covered by B.S. 3921, with specific requirements on water absorption frost resistance and soluble salt content. The materials for the mortar are also covered by British Standards, and the mix itself is controlled by a performance specification.

More difficult is the control of the resulting brick/mortar combination, which depends to a large extent on the skill of the bricklayer. To try to achieve some works control, 9 in. $\times 9$ in. ( $230 \mathrm{~mm} \times 230 \mathrm{~mm}$ ) brick cubes are made with six bricks in three courses (Ref. 8). Failure usually results from vertical splitting through the bricks and crushing at the corners, which are the least restrained.
In the laboratory, there is a good correspondence between the strength of such cubes
and the strength of $41 / 2 \mathrm{in}$. ( 115 mm ) walls built with the same materials. Various attempts have been made to relate the strength of brick walls to the strengths of their constituent bricks and mortar and an empirical formula has been suggested:
$\sigma_{\text {Brickwork }}=0.45\left[\left(\sigma_{\text {Brick }}\right)^{2} \times \sigma_{\text {Mortar }}\right]^{1 / 2}$
The standards of construction specified are similar to those in the concrete specification with tolerances of $+1 / 8 \mathrm{in}$. ( 3 mm ) for dimensions less than $10 \mathrm{ft} .(3 \mathrm{~m})$ on setting out. $1 / 4 \mathrm{in}$. ( 6 mm ) out of plumb in any storey height and $1 / 2 \mathrm{in}$. ( 13 mm ) out of plumb in the full height. Each wall has one face built smooth and this face is used for all subsequent measurements. The thickness of mortar beds is controlled by the requirement that the wall shall rise 1 ft . ( 300 mm ) in four courses where the standard $25 / 8 \mathrm{in}$. ( 67 mm ) bricks are used and all mortar joints are required to be well filled. For the bricklaying trade this means virtually the same standard of work as for facing brickwork, except that no pointing is required.
The protection of brickwork against frost damage is more difficult than for concrete, as the ratio of surface area to volume in brickwork is much higher than for most concrete sections and there is no shuttering to provide protection. As in winter concreting, the materials used can all be heated and the finished brickwork can be covered with protective sheeting. However, unless one builds in a complete enclosure, the mortar becomes cold while standing and one cannot cover the wall on which a bricklayer is actually working. We therefore specified that bricks should not be laid when the temperature was below freezing.

## Services

Where chases and holes for services were required, we could not allow the heating and electrical sub-contractors their usual freedom in cutting them. Only vertical chases were permitted to accommodate conduits and back-to-back chases were not allowed. All holes through the walls were required to be formed during construction, and the sizes and positions of all chases and holes had to be approved by us prior to construction. This required the sub-contractors to produce builders' work information much earlier than usual, but this was made easier by the negotiated contract.

## Pre-contract preparations

As the contractor was known for about a year before the start of the work on site we had adequate time to find suitable sources for supply of materials. The brick which we approved for use throughout was an Auchinlea 3 hole perforated, which had an average crushing strength of 5000 p.s.i. $\left(34.5 \mathrm{~N} / \mathrm{mm}^{2}\right)$ The contractor had some difficulty in finding a suitable brick as the quality of bricks generally available in Scotland is lower than in England. A Dewer \& Finlay Grade A Engineering brick was used in the 6 in . ( 150 mm ) brick-on-edge walls to give the necessary strength when loaded on edge.
The first samples of sand submitted gave very poor mortar strengths due to the sand being too fine. Strong mortar requires a sand at the coarse end of the range given in B.S. 1200 , Table 1, and in fact a sand in accordance with

Table I: Sand Grading For Mortar (1:1/4:3 Nominal)

|  |  | \% Passing By Weight |  |
| :--- | ---: | ---: | :--- |
| B.S. Sieve | B.S. 1200 | B.S. 1200 | Sand Approved for |
|  | Table 1 | Table 2 | ECODS. Clermiston. |
| $3 / 18$ in. | 100 | 100 | 100 |
| No. 7 | $90-100$ | $90-100$ | 100 |
| No. 14 | $70-100$ | $70-100$ | 98.5 |
| No. 25 | $40-100$ | $40-80$ | 90.0 |
| No. 52 | $5-70$ | $5-40$ | 47.4 |
| No. 100 | $0-15$ | $0-10$ | 6.0 |

## Table II: Works cube test results

Foster grade 6000 bricks used for this phase of contract. All values are in lbs./sq. in. $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$

|  | Mean | Range | Standard <br> Deviation |
| :--- | :--- | :--- | ---: |
| Mortar 7 day <br> ( 4 i c. cubes) | $2100(14.48)$ | $700-3500(4.83-23.15)$ | $850(5.86)$ |
| Mortar 28 day <br> (4 in. cubes) | $3000(20.7)$ | $1250-4200(8.62-28.96)$ | $1000(6.89)$ |
| Br. wk. 28 day <br> ( 9 in. cubes) <br> Br. wk. 28 day <br> ( 9 in. cubes) | $1950(13.44)$ | $1300-3100(8.96-21.37)$ | $500(3.45)$ |

Table 2 would be better. The respective gradings are given in Table 1 with the grading of the sand finally approved. It is easier to obtain a suitable sand in Southern England than it is in Scotland, where all sands tend to have a high proportion of fines.
The average 28 -day strength of the preliminary cubes manufactured at Edinburgh University from the approved sand and bricks was approximately 2500 p.s.i. ( $17.25 \mathrm{~N} / \mathrm{mm}^{2}$ ) for the mortar and 2600 p.s.i. ( $17.93 \mathrm{~N} / \mathrm{mm}^{2}$ ) for the 9 in . ( 230 mm ) brick cubes.
We were fortunate that the clerk of works appointed for the project was a stone-masor by trade and he introduced us to the practicalities of bricklaying. He was familiar with the local labour force and their standards of work, and proved a valuable help in the application of the specification and the maintenance of the quality of workmanship.

## Work on site

At the time of writing (May 1969). Block A is complete, Block B is nearing completion and Block C is at ground level. About $60 \%$ of the total brickwork has been built.
By the time bricklaying commenced in May 1968, there had been two amendments to materials previously approved. First, the contractor proposed using ready-mixed lime and sand, as it was easier to handle on site. However, this would have required a complete repetition of the preliminary brick cube and mortar cube tests because of the different sand source. He therefore proposed the use of a plasticizing agent. Febmix, instead of lime, which was accepted. It is an air entraining agent added to the mixing water, making the mortar more workable and also improving its frost resistance. Bricklayers sometimes use household detergent for the same purpose, but one has more confidence in the proprietary product !
The second change arose because Auchinlea bricks were no longer manufactured $2 \%$ in. ( 67 mm ) thick. Most bricks made in Scotland are $2 \% / \%$ in. $(73 \mathrm{~mm})$, but these could not be built to course with the Fyfestone on the gable walls. The first bricks delivered to site, therefore, were Foster perforated semiengineering bricks, manufactured in the north east of England and having a crushing strength of about 6000 p.s.i. ( $41.4 \mathrm{~N} / \mathrm{mm}^{2}$ ). Later the contractor requested a change to a cheaper Scottish brick which could be obtained $2 \% / \mathrm{in}$. ( 67 mm ) thick. This was a Darngavil solid brick which has a crushing strength of just under 4000 p.s.i. (27.6 $\mathrm{N} / \mathrm{mm}^{2}$ ). We agreed that this brick could be used on the upper floors of all three blocks, but that the Foster bricks would be used at all ground floor levels.
The first control cubes taken gave very low mortar strengths because of inadequate control of the water content, as the mortar had been mixed to suit the bricklayers. Some more cubes were then made with the mortar as stiff as was acceptable to the bricklayers. Satisfactory results were obtained and this quantity of water was used in all subsequent mixes. A proposed method for testing the consistency of mortar, equivalent to a slump test, is to drop a steel ball from a
known height into a sample of the mortar, but this is only at the research stage. In addition, there is far less information about water/cement ratios for mortar than for concrete.
There were some labour troubles initially as the bricklayers were having to work to facing brickwork standards at common brickwork rates. This led to a rather high turnover of bricklayers, which meant that each new squad had to be made familiar with the requirements of the specification. Several walls built by squads who were only on site for a short period had to be condemned for being outside the tolerances specified. Later the labour force settled down and the quality of the work improved greatly. Test cubes were taken fairly frequently at first, but are now taken every 200 square yards ( $167 \mathrm{~m}^{2}$ ) of brickwork as specified.
As could be expected, there has been a much greater range of results, both for the mortar and the brick cubes, than is normal with concrete. The range in mortar strengths is probably due to variations in water content and also inaccuracies in gauging materials. Care is always taken to ensure that mortar cubes are made from the same batch as is used for making the brick cubes. Large variafions have also occurred in the brick cube strengths within one set and there has been little correlation between the brick cube and the mortar cube results. The brick cube strengths depend on three things: brick strength, mortar strength and workmanship (Ref. 8). The effect of the latter would appear to be masking most of the effects of the other two. Table II gives the mean, the range, and the standard deviation of the brick and mortar cube results to date, all made with the Foster bricks. It also gives the specified preliminary and works strengths.
The Foster bricks are machine made and wire cut with the perforations giving an even firing, so that they are very consistent dimensionally. In comparison, the Darngavil bricks are made in moulds and fired in the traditional kiln, so that they tend to vary in size. The first deliveries of the Darngavil bricks were very poor in this respect and we had to return some to the manufacturer. His reaction was that nobody had complained in twenty-five years, but he did improve the quality and subsequent deliveries were quite good.
The provisions for services mentioned at the design stage required careful supervision on site. Chases were formed by cutting two slots with a carborundum disc and removing the material between with a hammer and chisel. Inevitably there were requests for revisions to chases which had already been formed. These were not permitted if the new position was close to the previous one as the effective length of the walls would have been reduced. So far only a few days work has been lost due to cold weather. On several days, bricklaying has not been able to start until the afternoon. This is partly due to B.S.T., but at least the cold does not set in again until an hour later in the afternoon. Salamander-type heaters are usually placed in the areas of work, largely to keep the bricklayers them-
selves warm, but the brickwork benefits as well. All fresh brickwork is covered with polythene sheets and the heaters are left on overnight. So far there is no equivalent to the Schmidt hammer test for assessing the actual strength of the mortar as built. A method using pins fired into the mortar bed joints has been proposed, but this would probably be even less accurate than the Schmidt hammer test.

## Conclusions

As mentioned in the introduction, loadbearing brickwork can be very economical. The comparative prices of walls on this project are given in Table III. Although the brick-on-edge walls are of comparable price to concrete, they have the advantage of using one trade to build all walls. The price for the $4 \frac{1}{2}$ in. $(115 \mathrm{~mm})$ brickwork is only slightly above that for common brickwork and does not perhaps fully reflect the better standard of workmanship.

| Table III: Comparative costs |  |  |
| :--- | :--- | :--- |
|  | Cost of <br> bricks per <br> thousand | Cost of <br> wall per <br> sq. yd. |
| 41/2in. brickwork | $235 /-$ | $30-35 /-$ |
| 9 in. brickwork | $235 /-$ | $60 /-$ |
| 6 in. 'brick-on-edge | $600 /-$ | $90 /-$ |
| 6 in. concrete | - | $80-90 /-$ |
| (nominal |  |  |
| reinforcement) |  |  |

For those considering using brickwork as a structural medium, these are some points which should be borne in mind.

1. When designing, keep to the fewest possible brick/mortar combinations.
2. Avoid short lengths of wall. They have little strength and would be best made nonloadbearing.
3. Ensure that the contractor appreciates what he is taking on at the tender stage.
4. Check on the proposed sources of supply of bricks and sand to ensure that they will continue for the duration of the contract.
5. It is a great advantage if the contractor has an established bricklaying squad who will stay for the duration of the contract and will, therefore, be familiar with the requirements of the specification.
6. Talk to the bricklayers yourself to ensure that they appreciate what is expected of them. The strength of the finished work is more dependent on the human element than is the case with concrete.
In addition, the following points should be covered more rigorously in any new specification.
7. Sand should be to B.S. 1200 Table 2.
8. Stricter control of the mortar mixing process should be specified. Mortar is usually not required in large enough quantities for the use of a weigh-batching mixer so the use of gauging boxes for sand, cement and lime should be obligatory, as should the use of measuring cans for the water.
9. The use of 9 in . ( 230 mm ) brick cubes would not appear to be of much value in assessing the strengths of the walls as built. However, they probably do have a beneficial psychological effect on the bricklayers and perhaps the engineer as well.
10. The precautions to be taken for cold weather working should be set out in more detail and the contractor should be required to submit his proposals to satisfy these requirements as part of the contract documents.
11. Specifications of bricks by tensile strength. Tests on full size walls show that under direct vertical compression, failure results from vertical splitting of the walls, with the cracks
running straight through the mortar joints and the bricks themselves. Research is being carried out at the moment on the tensile strengths of individual bricks and their relation to the strength of a whole wall. When this work is complete it may become necessary to specify bricks by their tensile rather than comprehensive strength.

## Specification requirements

1. Specified preliminary mortar 7 day cube strength: 1600 p.s.i. ( $11.03 \mathrm{~N} / \mathrm{mm}^{2}$ )
2. Specified preliminary mortar 28 day cube strength : 2400 p.s.i. ( $16.55 \mathrm{~N} / \mathrm{mm}^{2}$ )
Average value obtained in laboratory: 2500 p.s.i. ( $17.25 \mathrm{~N} / \mathrm{mm}^{2}$ )
3. Average 28 day strength of 9 in . brickwork cubes obtained in laboratory using Grade 5000 Auchinlea brick: 2600 p.s.i. (17.93 $\mathrm{N} / \mathrm{mm}^{2}$ )
4. Predicted values of brickwork preliminary strength from formula (See p.28), using a mortar with a specified preliminary strength of 2400 p.s.i. ( $16.55 \mathrm{~N} / \mathrm{mm}^{2}$ ) at 28 days. Using Grade 4000 brick:

1520 p.s.i. $\left(10.48 \mathrm{~N} / \mathrm{mm}^{2}\right)$ Using Grade 5000 brick

1760 p.s.i. $\left(12.13 \mathrm{~N} / \mathrm{mm}^{2}\right)$ Using Grade 6000 brick

2000 p.s.i. $\left(13.8 \mathrm{~N} / \mathrm{mm}^{2}\right)$
5. Works strengths are specified to be not less than $2 / 3 \times$ preliminary strengths.

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Fig. 5
View of west wing of A block. (Photo: Peter Carr)

Fig. 6
A block. Gable view of west wing and side view of other wing. (Photo: Peter Carr)


# The <br> analysis of hanging roofs 

## Alistair Day and James Bunce

The use of flexible cables as the main load carrying elements in a structure is by no means a new concept; suspension bridges spring to mind as some of the earliest examples. Interest in this form of construction for roofs has grown in the last few years and. in fact, the firm has been concerned with the design of a number of these structures.
It is often desirable and sometimes essential to be able to predict accurately the forces and displacements in any structure and this is particularly true of suspension structures. The usual stiffness equations are often not suitable for these structures as Peter Rice discovered in connection with the analysis of one of the hanging roofs in the Mecca project. This article describes a simple but exact technique (dynamic relaxation) for the analysis of such assemblies.


Fig. 1
Elementary suspension structure.
The method is best explained by way of a very simple example :
Consider the structure shown in Fig. 1. This is the most elementary kind of suspension structure.
In the deformed state the following relationships hold:-
$T=\frac{E A}{L} e=\frac{E A}{L}\left[\left\{(V+y)^{2}+H^{2}\right\}^{1 / 2}-\left\{V^{2}+H^{2}\right\}^{1 / 2}\right]$

$$
\begin{equation*}
t=T / L+e \tag{1}
\end{equation*}
$$

where $\frac{E A}{L}=$ stiffness of each bar
$T=$ force in each bar
$t=$ tension coefficient in each bar
$\mathrm{e}=$ elongation of each bar
$y=$ vertical deflection of load
for equilibrium we have :-

$$
\begin{equation*}
P-2 t(V+y)=R=0 \tag{3}
\end{equation*}
$$

Before the structure attains this equilibrium it has to move under the action of the imposed force $P$. The governing equation of this motion can be written as :-

$$
\begin{equation*}
\mathrm{R}=\mathrm{M} \ddot{\mathrm{y}}+\mathrm{K} \dot{\mathrm{y}} \tag{4}
\end{equation*}
$$

where $M=$ mass concentrated at node
$\mathrm{K}=$ coefficient of viscous damping.
Equation 4 can be written in finite difference form as :-

$$
\begin{align*}
R_{j+1 / 2} & =\frac{M}{\triangle t}\left(V_{j+1}-V_{j}\right) \\
& +\frac{K}{2}\left(V_{j+1}+V_{j}\right) \tag{5}
\end{align*}
$$

solving this equation for $\mathrm{V}_{\mathrm{j}}+1$ we get :-

$$
\begin{aligned}
v_{j+1} & =\left[v_{j}\left(\frac{M}{\Delta t}-\frac{K}{2}\right\} /\left\{\frac{M}{\Delta t}+\frac{K}{2}\right\}\right] \\
& +\left[R_{j+1 / 2} /\left\{\frac{M}{\Delta t}+\frac{K}{2}\right\}\right]
\end{aligned}
$$

where $\Delta t=$ time increment
$\mathrm{V}_{\mathrm{j}}=$ velocity of mass at beginning of tıme interval.
$\mathrm{V}_{\mathrm{j}+1}=$ velocity at end of time interval.
$R_{j+1 / 2}=$ residual force at midpoint of time interval.


Fig. 2
Prestressed cable truss.


Fig. 3
Load displacement versus time.

Now since distance $=$ velocity $\times$ time we can write :-

$$
\begin{equation*}
v_{j+1 / 2}=y_{j+1 / 2}+v_{j+1} \Delta t \tag{7}
\end{equation*}
$$

where $y_{j+1 / 2}=$ displacement at midpoint of time interval.
$\mathrm{y}_{\mathrm{j}+1 / 2}=$ displacement at midpoint of next time interval.
Inspection of the above equations will reveal that a cycle of calculations can be carried out as follows:-

1. set $\mathrm{y}=0, \mathrm{v}=0, \mathrm{t}=0$ or $\mathrm{t}=\mathrm{t}_{1}$ where $t_{1}=$ initial prestress.
2. calculate R from Equation 3
$\checkmark$ from Equation 6 y from Equation 7
3. calculate $T$ from Equation 1 and $t$ from Equation 2
4. goto 2 .

A plot of displacement versus time will show that the mass oscillates about its position of equilibrium. This oscillatory motion is a characteristic property of the system and it depends upon the mass and member stiffness. In the absence of any damping force (i.e. $K=0$ ) the oscillatory motion will continue indefinitely. However, if damping is present, the amplitudes will decay progressively, and if the damping exceeds a certain critical value the oscillatory character of the motion will cease altogether.
It is a straightforward matter to generalize all the equations developed above for any two or three dimensional pin-jointed assembly: it is not possible, however, to include this generalization in an article of this length. A computer program using these generalized
equations has been developed by the writers and has been used to solve the two following examples:

1. Fig. 2 illustrates a scale model of a typical two-dimensional cable truss. An initial pretension coefficient of 10 lbf per inch was assumed in all the cable links. A satisfactory solution was obtained after 2000 iterations. The result obtained is shown in Fig. 3. The time increment and damping factor chosen in this problem were 0.003 seconds and 0.8 respectively. A better choice of damping factor would have resulted in a considerable reduction in the number of iterations required. Ideally the curve shown in Fig. 3 (which is obviously underdamped) would have only three stationary points, the first being a maximum, the second a minimum and the third would be acceptably close to the equilibrium configuration.
2. Fig. 4 shows a simple pendulum in its initial and final configurations. This problem is of course trivial but problems of this type where (relatively) enormous mechanical movements take place are very severe tests of any non-linear deformation theory. We found that this problem was solved satisfactorily using the program outlined above.
The method of analysis presented above appears to have a number of advantages over existing non-linear theories :-
a) The mathematics involved is very simple.
b) No assumptions with regard to smallness of strain, etc. are made.
c) The method is very easy to program and uses only a small number of computing instructions. All the above calculations were


Fig. 4
Simple pendulum problem.
carried out on an IBM 1620 computer. It is worthwhile noting that such a small computer could not solve even the simplest problem using orthodox methods which either explicitly or implicitly minimize the total potential energy.
d) In addition to the static analysis obtained, a non-linear dynamic analysis is included as a free bonus.

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CREDITS
Figs. 1, 2 and 4. Drawn by: Jim Bunce.
Fig. 3. Drawn by: Susan Pickard.


